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Article

On Formally Undecidable Propositions of Law: Legal Indeterminacy and the Implications of Metamathematics

by
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and
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I. Introduction

The Critical Legal Studies (CLS) movement has for several years criticized mainstream legal scholars' belief that "the law" can (and does) determine outcomes in legal disputes. Critical scholars charge that this vision of the law is an illusion; "legal formalism" does not exist. Judges are always free to make decisions—to determine outcomes—as they see fit, relying on their own predilections, insights and life experiences.

The Legal Realists of the 1920s and 1930s, of course, made similar arguments about indeterminacy in the law. Their challenge, however, was more practical than theoretical in nature.

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2. See Mark V. Tushnet, Perspectives on Critical Legal Studies, 52 GEO. WASH. L. REV. 239, 239 (1984) ("First, critical legal studies is a sustained attack on all types of formalism. Second, that attack is also directed towards liberal political theory because some version of formalism is essential to the coherence of liberal political theory.").


4. See John M. Farago, Intractable Cases: The Role of Uncertainty in the Concept of [1439]
ies movement, in contrast, offers a thorough theoretical deconstruction of contemporary American legal thought.\(^5\) Critical scholars charge that the law fails—not only on an empirical level, but also in the abstract—to achieve any measure of formalism. Simply put, the law cannot determine cases. The law is, and can only be, indeterminate.\(^6\)

To support this argument, Critical scholars have offered extensive commentary on the limitations of human language.\(^7\) Language is subjective and imprecise, they argue, traits which the law (being dependent on language) must share. Given the inherent limitations of language, the

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\(^5\) Hutchinson & Monahan, supra note 1, at 199 ("[CLS] adherents do not simply contest the practical policies yielded by traditional legal theory; they reject the very basis of the theory itself."); see also Minda, supra note 3, at 640 ("CLS . . . practitioners have developed a theoretical critique which ‘transcends’ law in its attempt to demonstrate the ‘politics of reason.’"). Of course, Critical scholarship’s divergence from mainstream thought involves more than an argument over formalism:

The difference between Critical and mainstream legal thought is that although the latter rejects formalism, it persists in the view that some viable distinctions can be drawn between legal reasoning and vulgar political debate. CLSers, on the other hand, refuse to hedge on the indeterminacy of the legal order. They view the attempt to tread some middle path as a desperate, face-saving effort to conceal the irremediable crisis within the legal process and the breakdown of the social order.

\(^6\) See Joseph W. Singer, The Player and the Cards: Nihilism and Legal Theory, 94 Yale L.J. 1, 5-6 (1984). As Professor Singer states:

The issues raised by the Critical Legal Studies movement have brought nihilism to center stage. Those of us associated with Critical Legal Studies believe that law is not apolitical and objective: Lawyers, judges, and scholars make highly controversial political choices, but use the ideology of legal reasoning to make our institutions appear natural and our rules appear neutral. This view of the legal system raises the possibility that there are no rational, objective criteria that can govern how we describe that system, or how we choose governmental institutions, or how we make legal decisions. Critical Legal Studies thus raises the specter of nihilism.

law cannot uniformly and objectively inform and constrain judicial decisions.⁸

Recently, mathematical theories and proofs have surfaced in debates over indeterminacy in the law.⁹ Professor Anthony D’Amato, for example, has argued that two related—though distinct—mathematical results, the Löwenheim-Skolem Theorems¹⁰ and Gödel’s Incompleteness Theorem,¹¹ demonstrate that even if language is objective and precise, the law still must prove inherently indeterminate. The Löwenheim-Skolem Theorems, according to Professor D’Amato, reveal that a “‘case’ can be decided either way consistent with any legal theory.”¹² Further, Gödel’s Incompleteness Theorem proves that “legal, textual, and linguistic demonstrations” must propagate “propositions (actually, an infinity of them) that can neither be proved nor disproved.”¹³ Because plausible legal theories support any position in any case, and because an infinite number of indeterminate legal propositions exist, legal formalism is an impossibility.

Professor Ken Kress, in contrast, challenges the application of mathematical proofs to the American legal system, and consequently their usefulness in debating legal formalism.¹⁴ He argues that certain mathematical proofs, like those of Löwenheim and Skolem, “will not ‘go

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⁸ See infra Part III.A.
¹⁰ See infra notes 198-203 and accompanying text.
¹¹ See infra Part III.B.2.
¹² D’Amato, Can Legislatures Constrain?, supra note 7, at 597 n.96; see also D’Amato, Pragmatic Indeterminacy, supra note 9, at 175-76 & nn.91-92.
¹³ D’Amato, Can Legislatures Constrain?, supra note 7, at 597; see also Farago, Intractable Cases, supra note 4, at 226-29 (arguing that law “is vulnerable to reasoning similar to Gödel’s,” so that it “will need, in essence, an infinite set of axioms not derivable from any finite starting point”).
through’ in legal English.” 15 Further, Professor Kress asserts that
because the technical meanings of mathematical terms “diverge substan-
tially” from “their legal homonyms,” drawing analogies and conclusions
from mathematics is “a fallacy of equivocation.” 16

In a different vein, Professors John Rogers and Robert Molzon pre-
sume that Gödel’s proof is germane to the law, 17 but attempt to diminish
the proof’s relative importance. 18 Specifically, they assert that even
though Gödel’s legal analogue means that the law is indeterminate, the
conclusion that “any result is possible” does not necessarily follow. 19 In-
stead, at least in the context of constitutional interpretation, Rogers and
Molzon argue: “If an interpretive technique is sufficiently indeterminate
to permit ten or twelve interpretations of a particular clause, that still
leaves millions of interpretations that are not permitted.” 20 They thus
conclude that Gödel’s Theorem has only a limited impact upon the law.

Mathematical principles offer a host of information that might prove
inspiring for the law. The premier question is whether these principles
“go through” when words and legal logic are exchanged for numbers and
mathematical operations. The purpose of this Article is to explore the
legal applications of Gödel’s Incompleteness Theorem 21 and the proofs of
Löwenheim and Skolem 22 in an attempt to assess their impact on legal
formalism.

15. Id. at 144.
16. Id. at 145.
17. Rogers & Molzon, supra note 9, at 992 (“Gödel’s theorem strongly suggests that it is
impossible to create a legal system that is ‘complete’ in the sense that there is a derivable rule
for every fact situation.”). In assuming that Gödel’s Theorem applies to the law, Professors
Rogers and Molzon almost certainly fall prey to Kress’s “fallacy of equivocation.” See supra
text accompanying notes 15-16. They rely upon a brief excerpt from a single mathematical
reference, confusing its technical terms (e.g., “higher order languages”) with broader legal
homophones. See Rogers & Molzon, supra note 9, at 996-97 n.9 (Gödel’s theorem holds for
arithmetic and for higher order languages expressive enough to describe arithmetic.) (citing C.
Smorynski, The Incompleteness Theorems, in HANDBOOK OF MATHEMATICAL LOGIC 821,
825-29 (Jon Barwise ed., 1977)). The cited work uses the term “higher order language” in an
extremely technical sense that does not necessarily include English. There, it referred to a
class of formally defined languages whose quantifiers extend to properties of objects as well as
to objects themselves. Smorynski, supra at 826, 829-30. It would be as much an error for a
student of jurisprudence to construe this mathematical term informally as it would be for a
layperson to presume that the legal term “malice aforethought” requires contempt and
premeditation.
18. Rogers & Molzon, supra note 9, at 1005-09.
19. Id. at 1008 (emphasis in original).
20. Id. (emphasis in original).
21. See infra Parts III.B.2-4.
22. See infra Part IV.B.
Mathematical proofs and their applications are often misunderstood and misstated—by mathematicians as well as lawyers. For the latter the error is often compounded by legal training that encourages analogy. We as lawyers are eager to analogize our field of study to other disciplines because our methodology teaches us to do so. Unfortunately, analogies are sometimes false, leading to unsubstantiated assumptions and gainsay. For this reason, though we are willing to assume here that the law can be ideally structured to support formalism, we attempt to prove directly for the law what has been proved for mathematics.

Toward this goal, Part II of this Article explains formalism as commonly understood in the scientific and legal communities. Part III addresses the formidable question of whether legal formalism is feasible. We first examine here the very basic issue of whether human language, or better yet a subset known as "legal English," is objective enough to provide building blocks for a formal system. Tentatively assuming that legal

23. Consider the following remark:
Like all philosophical sceptical [sic] problems the issues raised by Gödel's theorems are pregnant with possibilities and fraught with dangers. Chief amongst the latter is an inevitable tendency to become distanced from the fons et origo of these developments. For it becomes ever more tempting and acceptable to rely on the findings of commentators who might themselves have based their readings on earlier summaries. To be sure, it is common practice to accept the verdict obtained by the experts in a field without inspecting their findings. But such custom presupposes concord.


24. Compare, e.g., D'Amato, Can Legislatures Constrain?, supra note 7, at 598 n.96 ("My point is that if the Skolem-Lowenheim result is possible in mathematics—a deductive, precise system—it is a fortiori possible in common-law adjudication.") and D'Amato, Pragmatic Indeterminacy, supra note 9, at 176 n.91 ("Only trivially can Lowenheim-Skolem or Gödel be true of English and not true of mathematics.") and Harold A. McDougall, Social Movements, Law, and Implementation: A Clinical Dimension for the New Legal Process, 75 CORNELL L. REV. 83, 89 n.36 (1989) ("[I]n mathematics[,] a system of explanation cannot be simultaneously complete and consistent. Ergo, there must be indeterminacy in any system of explanation.") and Rogers & Molzon, supra note 9, at 992 (concluding that Gödel's proof "strongly suggests" indeterminacy in the law) with Stuart Banner, Please Don't Read the Title, 50 OHIO ST. L.J. 243, 253 n.33 (1989) ("the analogy [between Gödel's theorem for mathematics and the law] cannot be carried too far") and M.B.W. Sinclair, Notes Toward a Formal Model of Common Law, 62 IND. L.J. 355, 363 n.33 (1987) (questioning "why we should think Godel's theorem relevant to [the common-law] system.").

25. See infra Part III.A.

26. Consider Professor Douglas Hofstadter's remark on assumed extensions of Gödel's Theorem: "It would be a large mistake to think that what has been worked out with the utmost delicacy in mathematical logic should hold without modification in a completely different area." DOUGLAS R. HOFSTADTER, GÖDEL, ESCHER, BACH: AN ETERNAL GOLDEN BRAID 696 (1979) [hereinafter HOFSTADTER, GOLDEN BRAID]; see also Girardeau A. Spann, Secret Rights, 71 MINN. L. REV 669, 698-99 n.58 (1987) ("The differences between formal logical systems and our less rigorous legal system are substantial enough that it is probably not useful to attempt a direct application of Gödel's theorem to legal or philosophical analyses.").
English can support such a system, we next address the precise meaning of Gödel's Theorem and how it speaks to mathematics. With this as our groundwork, we lastly address whether the limitations of formalism, as exposed by Gödel's Theorem, also apply to the law. We conclude that Gödel's Theorem does in fact have a legal equivalent—a conclusion drawn from an explicit proof, constructed in the style of Gödel's Theorem itself, which demonstrates the existence of an inherently indeterminate proposition about the law.

Part IV then confronts the significance of such a proposition. A legal equivalent of Gödel's Theorem means not only that law cannot be rendered consistent and complete, but also that it must contain an infinite number of undecidable propositions. As is true of mathematics, law is revealed as a wholly indeterminate system. Every case turns on human judgment and intuition. Given this specific result, the proofs of Skolem and Löwenheim are exposed as relatively unimportant for the law. Our conclusion in Part IV that they in fact have no legal equivalent is thus rendered largely irrelevant. Indeed, that the Löwenheim-Skolem proofs do not "go through" to the law says nothing at all about the prospects of legal determinacy. Instead, indeterminacy is revealed through direct proof analogous to Gödel's Theorem, with or without the assistance of the distinct proofs of Löwenheim and Skolem.

II. In Search of Legal Formalism

When used in legal discourse, the term "formalism" has sometimes taken on a pejorative connotation. Professor Frederick Schauer, for instance, notes that "'formalist' is the adjective used to describe any judicial decision, style of legal thinking, or legal theory with which the user of the term disagrees."27 Though Professor Schauer's observation is undoubtedly true, legal formalism also has a positive side. Indeed, for reasons discussed below,28 and notwithstanding its occasional condescending reference, formalism is seen as the guiding principle behind the law; formal reasoning is the hallmark of a good lawyer. Judge Richard Posner goes so far as to state: "It is a safe bet that a majority of legal professionals are formalists."29

Because the legal understanding of formalism sometimes strays from its technical meaning, we think it wise at this juncture to specifically define our terms and demonstrate what they describe. "Formalism," as

27. Schauer, Formalism, supra note 7, at 510.
28. See infra Part II.B.
commonly understood by mathematicians, is the belief that decisions can be completely and consistently rendered by way of a formal system. A "formal system," in turn, is a reasoning process designed to mechanically deduce truths from certain given assumptions, and consists of:

1. a definite description of symbols and syntax that allows propositions of the system to be expressed (that is, an objective language);\(^{30}\)
2. a set of axioms, defined as "a finite list of general propositions whose truth, given the meanings of the symbols, is supposed to be self-evident";\(^{31}\) and
3. a set of rules by which new propositions—theorems—may be inferred from axioms and established propositions.\(^{32}\)

A crucial characteristic of a formal system is that all of the system's rules must be defined solely in terms of the system's language, and then only in relation to the other rules within the system. The system's language must be the only mechanism for communicating the relationships among and between the rules of the system, and the rules themselves must be the source of the information communicated. Thus, the results derived from such a system depend solely upon the rules of the system and its initial axioms.

Consider a classical formal system such as arithmetic.\(^{33}\) Arithmetic's language includes certain well-defined operations such as equivalency (=), addition (+), subtraction (−), multiplication (×), and division (÷). Its proof rules implement certain logical operations, such as "if \(P\) and \(Q\) are both true then \(P\) is true," as well as basic applications of the arithmetical operations described above, such as "\(x + y = y + x.\)"\(^{34}\) These axioms and arithmetical operations provide the recipe for deducing theorems and truths about the system. Using only the language and rules of arithmetic, one can establish that "seven plus five equals twelve." No outside information is used, nor can it be if the system is to retain its formal character.

Formal proofs are algorithmic in nature. An "algorithm" is a finite systematic procedure by which a solution to a given problem is achieved. The procedure must terminate after a finite number of steps, and the

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31. Id. at 104.
32. Id.
33. See Hofstadter, Golden Braid, supra note 26, at 216-17; see also Farago, Intractable Cases, supra note 4, at 201 ("[Arithmetic], like all formal systems, is a small collection of initial assumptions (undefined terms and axioms) from which we can derive and thereby demonstrate infinitely many additional propositions (theorems).”).
34. Hofstadter, Golden Braid, supra note 26, at 216-17.
operation at each step of the procedure must be well defined. Of critical importance is that the decision to be made in any given step must—absolutely must—be definite, without deference to human judgment.

Arithmetical division of non-negative integers by positive integers illustrates an algorithm. Division is a systematic procedure for deciding how many times one number will go into another number and what remainder, if any, exists at the end of the procedure. It proceeds like this: Given a dividend $A$ and divisor $B$, while $A$ is larger than $B$, replace $A$ by “$A - B$” and keep count of the number of times the replacement occurs. When the replaced $A$ becomes smaller than $B$, stop the procedure. The number of times the replacement occurs is the quotient, and the last $A$ is the remainder. The procedure is well defined and finite, and will always yield the same result given the same input values for $A$ and $B$.

Because formal systems are algorithmic, their results are rigorously objective and mechanically verifiable. Consequently, if one is secure in the quality of the system’s language and the self-evidence of its axioms, one will also be secure in the objectivity of the system’s results. Any two individuals using the same proposition and the same formal system should concur as to the truth or falsity of a given formal proof of that proposition. Where the individuals differ, at least one of them can be seen, by an objective and mechanical process, to have made a mistake. If a grade school student learning her multiplication tables divides fifty-four by six, for example, and produces a quotient other than nine, the teacher can be confident that the student is wrong.

Of course, all of this supposes formalism in an ideal state. Formalism, remember, is the belief that a system will mechanically yield complete, consistent, and “correct” results. As will be demonstrated later, systems that are constructed to reveal only formal truths (i.e., “design-

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35. PENROSE, supra note 30, at 31. Penrose describes an algorithm as follows: “At each step it is perfectly clear-cut what the operation is that has to be performed, and the decision as to the moment at which the whole process has terminated is also perfectly clear-cut. Moreover, the description of the whole procedure can be presented in finite terms . . . .” Id. (emphasis in original).
36. See id. at 31-33.
37. See id. at 32-33.
38. Note that the algorithm only gives valid results for those input values that are within the scope of its formal definition. For example, it gives incorrect results when the dividend is negative. More interestingly, the algorithm gives no result at all when the divisor is zero: Because replacing $A$ with $A - 0$ leaves $A$ unchanged, the algorithm would proceed without end—never producing an answer.
39. At least, if the student is unambiguously directed to be working in base 10.
40. See infra Part III.B.2.
edly" formal systems) do not always succeed in their mission. As scientists and mathematicians well know, no assurance can be given that even a designedly formal system will always generate consistent and objective results, or any results at all, in determining the validity of a given proposition. Indeed, today when mathematicians refer to a given system as "formal" they implicitly recognize that the system must prove either inconsistent or incomplete.41

For instance, rather than asking two students to verify whether six goes into fifty-four nine times, suppose that the students are asked to independently prove or disprove, using arithmetic, Fermat’s Last Theorem.42 Here, the two students might reach different conclusions without either one of them having made an objective mistake—or more likely, they might be unable to come up with any answer at all. Hence, systems that are designed formally need not, by themselves, provide a result for every valid query.43 Moreover, a system’s capacity to mechanically validate an answer, once derived, does not assure that the system will always admit such an answer or that a contrary answer cannot also be obtained.

When a designedly formal system fails to render an objective answer to a problem, the system is said to be "indeterminate." Indeterminacy arises for two reasons. First, the system might be incomplete because its axioms and procedures do not admit any proof or disproof for a given proposition.44 If such is the case, the proof can be made only with insights gleaned from without the system. Because such extra-systemic insight is non-formal, the proof will be subjective, varying with the individual. That a system is formally designed does not guarantee that it will permit a proof or disproof for every proposition expressible in its language.

Second, a system could prove indeterminate because of an internal inconsistency.45 Its axioms (or combinations of axioms and proof rules)

41. See infra note 160 and accompanying text.
42. Fermat’s Last Theorem states that the equation $x^w + y^w = z^w$ does not hold for any integers where $x, y,$ and $z$ are greater than zero and $w$ is greater than two. The famous French mathematician Pierre de Fermat (1601-1665) made this assertion in the margin of Diophantus’s *Arithmetica*, but failed to offer a proof. See Penrose, supra note 30, at 58. To date, no one has proved (or disproved) Fermat’s Last Theorem.
43. Nevertheless, scientists and mathematicians continue to use such systems, for they remain useful, however imperfect.
44. See Douglas R. Hofstadter, Metamagical Themes: Questing for the Essence of Mind and Pattern 263-64 (1985) [hereinafter Hofstadter, Metamagical Themes]; cf. Singer, supra note 6, at 14 (“A legal theory or set of legal rules is completely determinate if it is comprehensive, consistent, directive and self-revising. Any doctrine or set of rules that fails to satisfy any one of these requirements is indeterminate . . . .”).
45. See Hofstadter, Metamagical Themes, supra note 44, at 263-65.
might prove contradictory and yield conflicting results. This can be especially debilitating to a formal system, because a single inconsistency can impeach the validity of every proposition within the system. In the propositional calculus,\(^46\) for example, every expressible proposition is derivable once another proposition and its negation have been proved.\(^47\) The ability to prove a single inconsistency would so infect the entire system that all propositions (and their negations) would be true.\(^48\)

Of course, inconsistency and incompleteness limit a designedly formal system’s value. Algorithms and formal systems are useful to scientists and mathematicians because of their objectivity. Determinate formal systems allow for precise predictions. Hypothetical data or data collected through observation are entered into the system, which can then be used to mechanically predict a future state of affairs. A determinate formal system insures that results will be consistent, complete, and “correct.” Indeed, formal results are “communicable”; any person (or machine) using the system can deduce them.\(^49\)

A. Comparing Legal Formalism

A determinate formal system of law would provide, for some, an ideal state.\(^50\) Rules and procedures would exist to determine algorithm-

\(^46\) The propositional calculus is a formal system for expressing compound and complex propositions and their logical relationships. \(9\) THE NEW ENCYCLOPAEDIA BRITANNICA 733 (15th ed. 1991). For an elaborate but informal description of the propositional calculus, see \textit{Hofstadter, Golden Braid, supra} note 26, at 181-97.

\(^47\) \textit{Hofstadter, Golden Braid, supra} note 26, at 195-97 (proof that in propositional calculus, “[f]rom a contradiction, everything follows”).

\(^48\) \textit{Id.} at 196. It should be noted that it is meaningless to state that two propositions are inconsistent until one imposes an interpretation upon them. \textit{Id.} at 94. For example, the propositions “\(1 + 1 = 2\)” and “\(1 + 1 = 3\)” need not be inconsistent, even if we give the numerals and plus symbol (“+”) their usual meanings; we might interpret “=” to mean “greater than or equal to.” It is only in light of a given symbolic interpretation that the notion of formal consistency has meaning.

\(^49\) \textit{Penrose, supra} note 30, at 417-18. Consider, for example, classical Newtonian mechanics. With the aid of formal mathematics, Sir Isaac Newton discovered that two bodies attract each other in proportion to the product of their masses and the reciprocal of the square of the distance between them (the “inverse square law”). \textit{Id.} at 166. Using Newton’s proposition (together with his derivative laws of motion), classical physical events can be accurately predicted. Because the description of physical events is contained in a formal system, it is objective and can be duplicated by anyone who knows the rules of the system.

\(^50\) See \textit{Ronald Dworkin, Taking Rights Seriously} (1977); Ronald Dworkin, \textit{No Right Answer?}, 53 N.Y.U. L. REV. 1 (1978). Professor Dworkin, perhaps more than anyone else, holds that law can be complete and consistent. With his “rights thesis” he argues that almost every case has a determinate answer. \textit{Dworkin, supra} at 82-90. Even when the case emerges as a “hard” one, Dworkin suggests that a formal procedure can be employed to reach the correct result. \textit{Id.} at 81; see also Richard H. Fallon, Jr. & Daniel J. Meltzer, \textit{New Law, Non-Retroactivity, and Constitutional Remedies}, 104 HARV. L. REV. 1731, 1759 (1991).
cally the outcome in any and every case. Parties \( A \) and \( B \) would have their dispute resolved in much the same way the number \( A \) is divided by the number \( B \). Each step would be well defined and definite and would eventually direct a judge to the proper, determinate result.

At the turn of the century and until roughly the 1920s, many legal scholars and jurists imagined that this perfect legal world actually existed.\(^5\) During this age of "conceptualism," disputes were supposedly resolved by "rule of law," the equivalent of what we have described here as a determinate formal system. Under this rule of law, there was no room for judicial intuition or discretion.\(^5\) In *Lochner v. New York*,\(^5\) for example, the Supreme Court declared (more or less) that socialism was unconstitutional because it conflicted with a specific rule of law, that is, "liberty of contract," required by the Due Process Clause of the Fourteenth Amendment.\(^5\) The Court pretended, or perhaps genuinely believed, that the result was somehow compelled by the Constitution.\(^5\)

*Lochner's* result was thus seen as a discovery and not a development.

Justice Holmes observed in his dissenting opinion in *Lochner* that the Constitution did not compel the Court to read the Due Process Clause to preclude social legislation\(^5\)—an opinion shared by followers of the Realist movement and most scholars today.\(^5\) Modern thought recognizes that the Court in *Lochner* was making policy rather than discovering the commands of the Constitution.\(^5\)

("Among contemporary jurisprudential writers, Ronald Dworkin might appear a spiritual heir to Blackstone. Like Blackstone, Dworkin depicts law as a seamless web, and he maintains that all legal questions have one right answer.").

\(^5\) See Hutchinson & Monahan, *supra* note 1, at 203.

\(^5\) Id.

\(^5\) 198 U.S. 45 (1905).

\(^5\) See *id*. at 53, 57-58, 61, 64.

\(^5\) See, e.g., *id*. at 56 (framing the issue as whether the challenged statute was "a fair, reasonable and appropriate exercise of the police power of the State [or] an unreasonable, unnecessary and arbitrary interference with the right of the individual" to freely make contracts); *id*. at 56-57 (denying that this involved "substituting the judgment of the court for that of the legislature"); *id*. at 61 ("We do not believe in the soundness of the views which uphold this law. On the contrary, we think that a law such as this . . . is an illegal interference with the rights of individuals . . . to make contracts . . . .").

\(^5\) 56. *Id*. at 75-76 (Holmes, J., dissenting).


\(^5\) See Schauer, *Formalism*, *supra* note 7, at 511 ("The formalism in *Lochner* inheres in
Clause offers only a vague command to the courts, using it to mechanically determine the outcome in any given case is impossible.59

Holmes’s specific objections in *Lochner* were foreshadowed by his more general comments about jurisprudence in *The Common Law*,60 where he wrote: “The truth is, that the law is always approaching, and never reaching, consistency.”61 Holmes further speculated in *The Path of the Law*62 that “[t]he language of judicial decision is mainly the language of logic. And the logical method and form flatter that longing for certainty and for repose which is in every human mind. But certainty generally is illusion, and repose is not the destiny of man.”63 Holmes thus illustrates that even during conceptualism’s heyday, skeptics questioned the law’s ability to achieve consistency and completeness.64

Notwithstanding Holmes’s reputation as perhaps the greatest jurist of modern times, mainstream scholarship continues to teach formalism within the law.65 The argument is that although cases impeaching the law’s completeness and consistency might arise, the vast majority of disputes have singular, determinate answers. The law provides solutions

its denial of the political, moral, social, and economic choices involved in the decision, and indeed in its denial that there was any choice at all.”).

59. For instance, as contrasted with the *Lochner* Court’s conclusion, Professor Mark Tushnet makes a persuasive argument that the Constitution compels socialism. See Mark V. Tushnet, *Dia-Tribe*, 78 Mich. L. Rev. 694, 696-701 (1979) (reviewing Lawrence Tribe, *American Constitutional Law* (1978)).


61. Id. at 36; see also Daniel Kornstein, *The Music of the Laws* 124 (1982).


63. Id. at 465-66.

64. Kornstein, supra note 61, at 128 (“Holmes’s creativity is shown by his perceiving, while focusing on law, the limitations that circumscribe any axiomatic and deductive system of a reasonable richness.”) (emphasis in original).

65. Although most lawyers, judges, and professors are willing to admit that the law is not technically, strictly formal, they continue to treat it as such. See, e.g., Posner, *Jurisprudence*, supra note 29, at 41 (noting that “a majority of legal professionals are formalists”); see also Rolf Sartorius, *Bayes’ Theorem, Hard Cases, and Judicial Discretion*, 11 Ga. L. Rev. 1269, 1269 (1977) (“There is a uniquely correct result in the vast majority of cases . . . .”); Schauer, *Formalism*, supra note 7, at 546 (“[L]et me satisfy myself here with the unproved empirical conclusion that linguistic instructions are sometimes potent. If such instructions sometimes create presumptions, and if those presumptions sometimes work, then what does this say about the possibility of what we might call a presumptive formalism.”) (emphasis in original). Courts offer conclusions as if no other result could be allowed and scholars write articles that purport to explain the law (as it is or as it should be). See, e.g., Wilson v. Seiter, 111 S. Ct. 2321, 2326 (1991) (“An intent requirement is either implicit in the word ‘punishment’ or is not; it cannot be alternately required and ignored as policy considerations might dictate.”); Donald H. Reagan, *The Supreme Court and State Protectionism: Making Sense of the Dormant Commerce Clause*, 84 Mich. L. Rev. 1091 (1986) (arguing that all modern Supreme Court cases invalidating state laws under the Commerce Clause can be singularly explained).
either by clear command or by way of legal "theory."66 "Legal formalism" thus instructs that the law almost always resolves societal disputes. Only in the rarest of cases must the judge look elsewhere for answers.67 And never must a judge flip a coin or guess at the proper legal result.68

Of course, a cursory empirical examination of Supreme Court decisions might lead one to question this assertion. The 1990 Term of the Supreme Court, for example, produced a plethora of divided opinions,69 many hopelessly so.70 If Justices of the Supreme Court, the most respected legal minds in the country, can only occasionally agree on the law's meaning, how can one credibly assert that the law is normally determinate?

66. For example, Judge Posner argues that simple economic theory explains many of the results obtained under the law. See RICHARD A. POSNER, THE ECONOMICS OF JUSTICE (1981). Professor John Hart Ely offers a process-oriented approach to explain the role of the judiciary in safeguarding the rights of individuals. JOHN HART ELY, DEMOCRACY AND DISTRUST (1980). Common to these theories is their implicit assumption that if used they can and do direct outcomes. They are attempts at achieving formality.

67. See Michel Rosenfeld, Deconstruction and Legal Interpretation: Conflict, Indeterminacy and the Temptations of the New Legal Formalism, 11 CARDOZO L. REV. 1211, 1233 (1990) ("[T]he new legal formalism is properly considered to be a type of formalism to the extent that it maintains that something internal to law rather than some extra-legal norms or processes determines juridical relationships and serves to separate the latter from non-juridical social relationships, including political ones.").

68. Of course, modern formalists must recognize the litigious nature of American society and the enormous number of suits currently pending in most courts' dockets. Arguably, this large number of cases supports the Critical claim of legal indeterminacy. One response to this observation might be that the large amount of litigation in the United States is not so much a function of indeterminacy in the law as it is a result of factual uncertainty. The facts of most cases simply are not clear. Without clarity in the input—the facts of a case—the output cannot be accurately predicted. Hence, cases are frequently tried in order to resolve factual questions. Once discovery is complete and the trial is over, however, the law determines the outcome.

Even assuming that factual uncertainty accounts for much of the current litigation, certainly not all of it can be attributed solely to factual disputes. Many cases with stipulated or established facts are still contested, as indicated by the large number of appellate cases that do not delve into factual matters. In any event, factual uncertainty is not at issue in the current determinacy debate. Instead, the challenge raised by the Critical Legal Studies movement focuses on the legal rules that apply to established facts. Critical scholars charge that determinacy cannot exist even at this level. Hutchinson & Monahan, supra note 1, at 199; cf. Anthony D'Amato, The Ultimate Injustice: When a Court Misstates the Facts, 11 CARDOZO L. REV. 1313, 1347 (1990) (arguing that courts can manipulate facts just as easily as they can manipulate law).

69. Roughly three-quarters of the Court's 120 full opinions rendered during the 1990 Term were divided. "Divided" cases are those in which either a dissenting opinion or a concurring opinion that disagreed with the Court's reasoning was filed. See The Supreme Court, 1990 Term—The Tables, 105 HARV. L. REV. 419, 421 (1991) (Of 120 full opinions, 85 included either dissents or concurrences disagreeing with the majority's reasoning).

70. In addition to several plurality opinions, the Court rendered twenty-one 5-4 decisions during the 1990 Term. See id. at 422.
Professor Schauer has responded to this particular argument by noting the discretionary nature of the Supreme Court's jurisdiction. Because the Justices choose only to hear difficult, indeterminate cases, Professor Schauer argues, "the Supreme Court, far from being the first place to look for easy cases, ought to be the last place." Moreover, the bulk of the Court's docket consists of constitutional cases in which the open-ended language of the Constitution more readily gives rise to dissent.

Lower courts, in contrast, are less divisive, especially where constitutional law is not at issue. Consensus over the outcome of legal issues is more common in state and lower federal courts, but unanimity is by no means common enough to allow a confident proclamation that the

71. Frederick Schauer, Easy Cases, 58 S. CAL. L. REV. 399, 409 (1985) [hereinafter Schauer, Easy Cases]. Prior to 1988, the Court was required pursuant to its appellate jurisdiction to hear only a small number of cases, including those where a state court had either struck down a federal statute or adjudged a state statute valid in the face of a possible conflict with federal law, see 28 U.S.C. § 1257(1)-(2) (1982) (amended 1988), those cases where a federal court of appeal had struck down a state statute, see 28 U.S.C. § 1254(2) (1982) (amended 1988), those cases to which a federal agency or officer was a party and any federal court had struck down a federal statute, see 28 U.S.C. § 1252 (1982) (repealed 1988), and those cases in which judgment was issued by a panel of three district judges under 28 U.S.C. § 2284, see 28 U.S.C. § 1253 (1988). Today, the Court's jurisdiction is almost completely discretionary, see 28 U.S.C. §§ 1254, 1257 (1988), these mandatory categories having been, except for the last mentioned above, eliminated by Act of June 27, 1988, Pub. L. No. 100-352, §§ 1-3, 102 Stat. 662, 662.

72. Schauer, Easy Cases, supra note 71, at 409; cf. Anthony D'Amato, Aspects of Deconstruction: Refuting Indeterminacy With One Bold Thought, 85 NW. U. L. REV. 113, 116 (1990) [hereinafter D'Amato, One Bold Thought] ("The Supreme Court, I contend, is no longer a court that decides cases. It has become in the last fifty years or so a legislative body which uses a case simply as a serendipitous vehicle for enacting social legislation.").

73. See D'Amato, One Bold Thought, supra note 72, at 114 (noting statistical studies which conclude that "dissents were filed in less than four percent of [appellate decisions in federal courts]") (citing Jon O. Newman, Between Legal Realism and Neutral Principles: The Legitimacy of Institutional Values, 72 CAL. L. REV. 200, 204 (1984) and Alvin B. Rubin, Doctrine in Decision-Making: Rationale or Rationalization, 1987 UTAH L. REV. 357, 367). Still, Professor Schauer admits that even in lower appellate courts, "there are few, if any, easy cases." Schauer, Easy Cases, supra note 71, at 411. See generally D'Amato, Pragmatic Indeterminacy, supra note 9, at 162-63 (discussing Schauer's contentions).

74. Schauer argues:

In [lower federal and state appellate courts], at least with respect to appeals of right, there are many cases that would be perceived by the court involved, the academic world, other external observers—indeed by everyone except the appellant—as easy. In these instances, claims are either upheld or denied on the basis of little more than mechanical application of existing rules with little anguish on the part of the court. See Schauer, Easy Cases, supra note 71, at 410 (footnote omitted). But see D'Amato, Pragmatic Indeterminacy, supra note 9, at 163 (observing that even in lower appellate courts plausible arguments exist for both sides in every case).
law is determinate. Perhaps the best that can be said is that in the appellate courts the law sometimes seems consistent and complete while at other times it appears confused and contradictory.

Explanations for occasional departures from the formal ideal can be readily imagined. One might argue that some judges (as well as lawyers and professors) simply are not as smart as others. And even intelligent judges make mistakes. Either of these contingencies could explain some of the judicial disagreement about the law.

Moreover, one might respond to the inconsistencies and dissenting opinions found in the various reporters by observing that reported cases represent only a very small percentage of the controversies and transactions that occur in the United States each day. The vast majority of transactions occur without incident, the argument goes, because the law informs people of their rights and obligations. Stores sell goods, for example, without ever having to resort to the courts. These "cases" never go to court because they are determinate.

Of those cases that do wind up in court, only a small portion are appealed and reported. Most cases are routinely resolved at trial, with little or no dispute about the proper rules of law. In only a very few cases is the law ever seriously contested, and these are the cases that work their way into the reports. Given the nature of these reported cases, it is not surprising that there should be dissent and inconsistency.

Apologetic arguments of this nature reflect a rather bald admission that the law is not truly formal. Formalism is not so loose a concept

75. Professor D'Amato concludes that a low rate of dissenting opinions tends to prove the law's indeterminacy rather than its determinacy:

Given the deconstructionist's view that law does not constrain a judge's ruling in any given case, there is little point in dissenting. A decision in any case is reached by the brute force of majority rule. The majority was not constrained by law to reach the decision it reached, as the minority well knows. Hence there is nothing to be gained by dissenting.

D'Amato, One Bold Thought, supra note 72, at 115.

76. See Rogers & Molzon, supra note 9, at 1000 ("[T]here are many misapplications of the law. The law may still be assumed consistent even though different judges make inconsistent determinations on indistinguishable facts.") (emphasis added).

77. See Schauer, Easy Cases, supra note 71, at 412 ("Every time some claimed grievance stays in the lawyer's office because litigation seems futile, we have an easy case.").

78. Id. But see D'Amato, Pragmatic Indeterminacy, supra note 9, at 166-71 (disputing that such cases are easy or determinate).

79. See Schauer, Easy Cases, supra note 71, at 411 ("Few cases that are filed reach final decision after a full evidentiary hearing. Many are settled, and many others are decided by the various devices designed to sort out the hard cases from the easy ones, particularly summary judgment and dismissals on the pleadings."). But see D'Amato, Pragmatic Indeterminacy, supra note 9, at 163 ("[N]o easy case is anywhere to be seen in trial court.").

80. Professor D'Amato notes that most legal scholars today attempt to "have it both
that it can only generally exist; either it does or it does not. Any accurate description of the American legal system must recognize that the law has yet to achieve determinate formality. This acknowledgement, however, does not answer the normative question of whether the law can operate in a determinate, formal fashion. Perhaps these imperfections in the law can be corrected. Maybe the current indeterminacy in the law is only contingent; practical difficulties such as imprecise drafting, miscommunication, or even simple human error may be the causes. Controlling for these practical contingencies, the question then is the normative one of whether the law—in theory—can be ideally formalized.

B. Why Aspire to Legal Formalism?

Before proceeding, one might inquire of the importance of legal formalism. The formal ideal has appeal to the legal system for at least three reasons. First, because of its objectivity, a determinate formal system would restrict the amount of discretion reserved to the judiciary and thereby promote the American vision of democratic government.

81. Compare Kenney Hegland, Indeterminacy: I Hardly Knew Thee, 33 ARIZ. L. REV. 509, 515-18 (1991) (arguing that some indeterminacy exists in the law) with D'Amato, Consequences of Plain Meaning, supra note 80, at 551 (Believing that the law is indeterminate but the world outside the law is determinate “is the same mental manipulation that the majority of legal scholars engage in today, when they say, along with Professor Ken Kress, that there is ‘some’ indeterminacy in the law but there is also a great deal of determinism.”).

82. See Robert H. Bork, Neutral Principles and Some First Amendment Problems, 47 IND. L. J. 1, 2 (1971) (“The requirement that the Court be principled arises from the resolution of the seeming anomaly of judicial supremacy in a democratic society. If the judiciary really is supreme, able to rule when and as it sees fit, the society is not democratic.”); Eric Rakowski, Posner's Pragmatism, 104 HARV. L. REV. 1681, 1682 (1991) (reviewing Posner, JURISPRUDENCE, supra note 29) (“But why should unelected judges' beliefs about morality or sound policy prevail in a democratic polity when they depart from the views of elected officials or the citizenry generally?”); Antonin Scalia, The Rule of Law as a Law of Rules, 56 U. CHI. L. REV. 1175, 1176 (1989) (“Statutes that are seen as establishing rules of inadequate clarity or precision are criticized, on that account, as undemocratic . . . .”).

83. Of course, this evolution took time, several constitutional amendments, and the assistance of the Supreme Court. See U.S. CONST. amend. XV (right to vote shall not be denied on account of race); U.S. CONST. amend. XIX (right to vote shall not be denied on account of sex); U.S. CONST. amend. XXIV (right to vote shall not be denied on account of failure to pay poll tax or other tax); U.S. CONST. amend. XXVI (right to vote shall not be denied to citizens that are 18 years of age or older on account of age); Kramer v. Union Free Sch. Dist., 395 U.S. 821 (1969) (holding unconstitutional a state law that restricted the right to vote in school board elections to land owners or lessors and parents or guardians of children enrolled in public schools); Harper v. Virginia Bd. of Elections, 383 U.S. 863 (1966) (holding
through their elected representatives, not from the judiciary. Making law is a job for the legislature. Entrusting the judicial branch with excessive discretion in interpretation and application of the law contradicts this republican ideal. Formalism in the law thus promotes the American political philosophy by strictly limiting judicial discretion.

State judges, of course, are sometimes elected and therefore can possess a measure of representative authority. Nevertheless, even elected judges are expected to decide cases in a determinate, formal fashion. Republicanism, therefore, cannot form the sole basis for aspiring to formality. And indeed it does not. Formalism has among its primary attributes certainty and reliability, qualities that endear themselves to the law both for their own sake and because they promote fairness and equality. Legal formalism allows people to accurately predict the legal consequences of their actions, which in turn fosters reliance and fairness within the American legal system. Perhaps more important, the certainty achieved through formalism insures that like cases are treated alike—thus assuring equal treatment under the law.

unconstitutional a state law that conditioned the right to vote on the payment of a tax); Reynolds v. Sims, 377 U.S. 533 (1964) (holding unconstitutional a state’s legislative apportionment system that diluted some citizens’ right to vote).

84. See U.S. CONST. art. IV, § 4, cl. 1 (guaranteeing to the states “a Republican Form of Government”).


86. See Chisom v. Roemer, 111 S. Ct. 2354, 2367 (1991) (“When each of several members of a court must be a resident of a separate district, and must be elected by the voters of that district, it seems both reasonable and realistic to characterize the winners as representatives of that district.”).

87. See Norman Barry, The Classical Theory of Law, 73 CORNELL L. REV. 283, 286 (1988) (“Law deals with the actions of private agents and has no purpose beyond providing a predictable framework for individuals to pursue their private ends with the minimum of collision with each other.”); Schauer, Formalism, supra note 7, at 539 (“One of the things that can be said for rules is the value variously expressed as predictability or certainty.”); see also John Finnis, Natural Law and Natural Rights 270-71 (1980) (discussing the value of reliability and predictability).

88. See Scalia, supra note 82, at 1179 (“[A]nother obvious advantage of establishing as soon as possible a clear, general principle of decision [is] predictability.”).

89. See id. at 1178 (“The Equal Protection Clause epitomizes justice more than any other provision of the Constitution. And the trouble with the discretion-conferring approach to judicial law making is that it does not satisfy this sense of justice very well.”); see also Anthony D’Amato, Can/Should Computers Replace Judges?, 11 GA. L. REV. 1277, 1300-01 (1977) [hereinafter D’Amato, Computers] (arguing that formalism would obviate judicial bias, enhance equality, and prove economically efficient); Vincent A. Wellman, Practical Reasoning
Finally, formalism tends to reinforce society's belief in the mystical "rule of law." Chief Justice Marshall stated almost two centuries ago in *Marbury v. Madison* that "[t]he government of the United States has been emphatically termed a government of laws, and not of men." Maintaining an objective system—one that is distinct from the people who administer it—is important because a detached legal system appears credible and perpetual. Regardless of the failings of those who manage the system, and notwithstanding the system's ever-changing personnel, the law retains respect and continuity. In short, formalism promotes the legitimacy of the American legal system.

Paradoxically, many mainstream scholars eschew formalism even while praising it. Professor Schauer, for example, answers the question, "[W]hat is so good about decision according to rules?" by flatly stating, "[N]othing." Judge Posner likewise questions the value of formalism: "Law can be highly objective and impersonal, yet thoroughly unjust." He accordingly proclaims: "I am not a formalist." And Norman Barry, a Professor of Politics at the University of Buckingham, offers a normative observation that "legal reasoning cannot be mechanistic or deductive."

Still, in law schools across the country and in virtually every courtroom, the formalist ideal remains paramount. Even Professor Barry, though renouncing "mechanistic" legal reasoning, argues that "[i]t is not the function of the judge to bring about some desirable state of affairs but to find *objectively* the right decision within the general system of rules, a system that exists independently of judicial activity itself." American culture demands determinacy and abhors license. Equity is the exception, not the rule. Instead, the rule is one "of law"—the law defined in determinate terms. Given that this ideal exists, we turn to the critical issue of whether it is necessarily an illusion.

*and Judicial Justification: Toward an Adequate Theory, 57 U. COLO. L. REV. 45, 55 (1985) ("[J]udges should treat like cases alike.").*
90. 5 U.S. (1 Cranch) 137 (1803).
91. *Id.* at 163.
92. Schauer, *Formalism, supra* note 7, at 539.
93. *Id.* Schauer also criticizes formalism by observing that "rules get in the way." *Id.* Schauer thus favors rules, but only those rules that do not constrain the decisionmaker too much.
95. *Id.* at 33.
96. Barry, *supra* note 87, at 285; *see also* Hegland, *supra* note 81, at 578 (finding a "formalist definition of justice" to be "repugnant").
98. Professor D'Amato has also offered his views on the costs of formalism. He observes that true formalism will (1) "freeze[... precedents"; (2) "render areas of the law uninterest-
III. Is Formal Law Possible?

A. The Problem of Linguistic Vagueness

As stated above, one requirement of a formal system is that its rules and syntax be certain and well defined.99 One who implements the system must not have room for judgment as to what the rules mean and how they are to be applied. In arithmetic, for example, one must know that "=" means equivalency. No room for disagreement on this point can exist, given the terms of the formal system.

A good deal of Critical scholarship has addressed whether language, and thus law, can be sufficiently defined so as to foster formalism. Professor Stanley Fish, for example, has argued that language is inherently subjective and thus cannot guide judges in any objective, neutral fashion.100 Judges are always free to decide cases notwithstanding the language of statutes and corresponding rules of interpretation.101

Mainstream scholarship’s rejoinder to Professor Fish’s argument has proceeded along the same lines as its response to empirical observations of indeterminacy in the law. The response may be paraphrased as follows: “True, language is occasionally vague, but normally it is not. Many words are certain and fixed, so that at least in regard to this large portion of language formalism can be achieved.”102 Professor Schauer, for example, has responded to Professor Fish’s argument by observing

99. See supra text accompanying notes 30-32.
100. See, e.g., STANLEY FISH, IS THERE A TEXT IN THIS CLASS? (1980); Fish, PLURALIST VISION, supra note 7, at 503 (“Any text, whatever its conditions of production, is capable of being appropriated by any number of persons and read in relation to concerns the speaker could not have foreseen.”); Fish, CHAIN GANG, supra note 7, at 564 (“The crucial point is that one cannot read or reread independently of intention, independently, that is, of the assumption that one is dealing with marks or sounds produced by an intentional being, a being situated in some enterprise in relation to which he has a purpose or a point of view.”) (emphasis in original); Fish, WRONG AGAIN, supra note 7, at 314 (“[I]t is precisely my thesis (with which of course one might quarrel) that in whatever way one establishes an interpretation, one will at the same time be assigning an intention.”); cf. J.L. AUSTIN, THE MEANING OF A WORD, IN PHILOSOPHICAL PAPERS 55-75 (J.O. Urman & G.J. Warnock eds., 3d ed. 1979) (explaining that words have no meaning outside the context of at least a sentence).
101. See, e.g., D'Amato, CAN LEGISLATURES CONSTRAIN?, supra note 7, at 562 (“My starting point, however—in alignment with Stanley Fish—is that there can never be a definitive theory or set of rules of interpretation.”).
102. See, e.g., Schauer, EASY CASES, supra note 71, at 418-20; Schauer, FORMALISM, supra note 7, at 512-14. Schauer thus attempts to marginalize the problems raised by linguistic vagueness:

More commonly, however, the indeterminacy to be filled by a decisionmaker's choice is not pervasive throughout the range of applications of a term. Instead, the indeterminacy is encountered only at the edges of a term's meaning. As H.L.A. Hart tells
that certain words have "inexorable" meanings: "When I say that pelicans are birds, the truth of the statement follows inexorably from the meaning of the term 'bird.' If someone disagrees, or points at a living, breathing, flying pelican and says 'That is not a bird,' she simply does not know what the word 'bird' means." Likewise, some rules are clearer than others, and certain cases are easier than others.

The stakes in the debate are high. If language is subjective, then law cannot be given formal structure. Two judges might come to different conclusions about a given rule's meaning because the words of the rule mean different things to each judge. The law could not function in an algorithmic fashion because of the vagueness presented by its language. In mathematical terms, it would be like trying to divide fifty-four by six, but allowing the meanings behind "fifty-four" and "six" to vary with the interpreter.

On the other hand, if language is objective, the law at least has the potential for being cast as a formal system. The primary question here is exactly how objective must language be. Mathematics makes use of a rigorous written language that is universally understood—at least by those who dabble in the field. This objective language then provides the framework for constructing formal mathematical models and systems. Can everyday language achieve this same level of certainty?

Nothing peculiar to human language would appear to preclude formalism within an appropriate interpretive community. Granted, because of language's richness and tendency toward ambiguity, it suffers a greater potential to develop subjective qualities than does arithmetic or calculus. As a practical matter, language might never overcome these

us, legal terms possess a core of settled meaning and a penumbra of debatable meaning.


103. Schauer, Formalism, supra note 7, at 512 (footnote omitted).

104. Professor D'Amato has responded cleverly to Schauer's example of an "inexorable" meaning by suggesting that "the 'that' in the speaker's sentence refers to her finger. . . . 'That' is a finger, she is asserting, not a 'bird.'" D'Amato, Aspects of Deconstruction, supra note 7, at 538 (emphasis in original). D'Amato concludes: "Language is so wonderfully variable, and its nuances so infinitely rich, that the notion of omitting all interpretations not imagined by a given law professor is worse than incoherent and worse than impossible—it is uninteresting." Id. at 540-41.

105. See Fish, Pluralist Vision, supra note 7, at 498 ("[T]here is nothing in principle to prevent the emergence of a unified legal interpretive community. . . . As an institution the law would then be in the happy state (if it is happy) enjoyed by certain branches of the physical sciences . . . .").
hurdles—thus leaving legal formalism as an impossible dream.106 But as an abstract matter, this need not be so.

The language of arithmetic can certainly be misunderstood or misinterpreted. Indeed, just as a person might misunderstand the word "bird," a person might subjectively develop her own meaning for the symbols of arithmetic. The numeral "2" might, for instance, mean seventeen to a particular person, making it impossible to communicate arithmetically with her.107 This possibility, however, does not destroy the usefulness of arithmetic as a formal system. It simply means that this unique individual is operating within a different system—one all her own—and that communicating with her might prove difficult.

Similarly, that people disagree over the meaning of a particular word should not be taken as an abstract proof of the limits of human language. Disagreement over meaning cannot be taken to demonstrate inherent subjectivity; rather it would appear to present more of an empirical observation. To date, nothing approaching a verifiable proof establishes that human language (and hence legal English) cannot form the basis for a formal, logical reasoning system. Such a system may be impossible, but that has yet to be determined. At least as far as the predicate language supporting the system is concerned, if mathematical symbols can succeed then so can legal English.108

Answers to this question might be uncovered by the emerging study of artificial intelligence (AI).109 Future advances in computer programming and expert systems surely will aid our understanding of human language and how it can be electronically "copied."110 Computers today

106. See Kress, supra note 14, at 142.

107. See generally SAUL A. KRIJKE, WITTGENSTEIN ON RULES AND PRIVATE LANGUAGE (1982). Kripke posits that a "bizarre skeptic" might argue that past understandings of algorithmic definitions cannot be objectively confirmed, even by recording former results. Id. at 8. What we now consider to be addition, we might formerly have believed to be "quaddition," which is identical to addition, except for some pair of numbers which we did not actually compute, say 68 and 57, for which the result is five. Id. at 8-9. Since we did not actually compute and record that particular sum in the past, the skeptic will argue there can be no proof that our past self did not formerly use quaddition instead of addition at that time. If this confusion can happen to a single mathematician, compared with his past self, it can happen between two mathematicians as well. Fortunately, Kripke proposes at least one solution to this skeptical problem. Id. at 107-09 (developing a variant of Ludwig Wittgenstein's "private language argument").

108. Other limitations besides vagueness may exist. In Part III.B, infra, we conclude that Gödel's Incompleteness Theorem applies to legal English and thus limits how legal English can be used.


110. Roger Penrose raises this point:
can perform myriad tasks that just a few years ago could only be performed by human beings. Software has been developed that allows machines to communicate with humans and assist in complex thought projects. For instance, computers can play (and win at) chess,\textsuperscript{111} and expert systems currently assist in “medical diagnoses, mathematics, meteorology, law, investment analysis, agriculture, and chemistry.”\textsuperscript{112} No doubt the years to come will witness even more “intelligent” programs that can assist in other human endeavors.

One of the more intriguing developments in the field of artificial intelligence is a general recognition that in the coming years, with increased storage space and greater processing speed, programs might be developed that simulate human thought.\textsuperscript{113} Even more fascinating (and disturbing to some) are the claims of a branch of AI research, called “strong AI,” which predicts that when computers can convincingly simulate human thought, they will have achieved consciousness.\textsuperscript{114} They will love and hate, experience pain and pleasure, just as humans do.\textsuperscript{115}

Also of interest commercially, as well as generally, is the development of expert systems, according to which the essential knowledge of an entire profession—medical, legal, etc.—is intended to be coded into a computer package. Is it possible that the experience and expertise of human members of these professions might actually be supplanted by such packages?

\textbf{Penrose, supra note 30, at 11 (emphasis in original).}

\textbf{111.} A program known as “Deep Thought” has gone so far as to defeat a Grandmaster. \textit{Penrose, supra note 30, at 13.}

\textbf{112.} Pamela Samuelson, \textit{Benson Revisited: The Case Against Patent Protections for Algorithms and Other Computer Program-Related Inventions}, 39 Emory L.J. 1025, 1115 (1990) (footnotes omitted). As for law, a computer program has been devised to assist lawyers with specific tax problems. \textit{See}, e.g., L. Thorne McCarty, \textit{Reflections on TAXMAN: An Experiment in Artificial Intelligence and Legal Reasoning}, 90 Harv. L. Rev. 837, 838 (1977) (“[T]he program . . . is capable of performing a very rudimentary form of ‘legal reasoning’: Given a ‘description’ of the ‘facts’ of a corporate reorganization case, it can develop an ‘analysis’ of these facts in terms of several legal ‘concepts.’ ‘’\textsuperscript{13}’’); \textit{see also} Rissland, \textit{supra} note 109, at 1961-81 (discussing uses of AI in the law).

\textbf{113.} \textit{See Penrose, supra note 30, at 17-18.}


\textbf{115.} The argument posits that the human brain is merely an organic microprocessor that continually runs algorithms. \textit{See Penrose, supra note 30, at 17 (“The idea [behind strong AI] is that mental activity is simply the carrying out of some well-defined sequence of operations, frequently referred to as an \textit{algorithm}.”) (emphasis in original). Human thought, and hence consciousness, is achieved through these algorithms. All emotions are achieved algorithmically—albeit in an extremely complicated fashion. \textit{Id.} Assuming electronic computers can achieve this level of complexity, they too will achieve consciousness—they will “feel” just as humans do. \textit{Id.} at 14 (“[S]ome AI supporters envisage that concepts such as pain or happiness can be appropriately modelled in this way.”).
To most people this proposition is absurd. How can machines think and feel, love and hate? The proposition not only appears religiously objectionable, but also belies common sense. Offering a reasoned explanation for these intuitive feelings, however, is a different matter. Indeed, explaining why machines cannot achieve consciousness is at least as difficult as explaining why they can.

Roger Penrose, a renowned mathematician and physicist at Oxford, offers a scientific rebuttal to strong AI. Penrose, in his book The Emperor's New Mind, explains that the universe itself is not determinate—or at least not in a way that we can understand and assimilate. As examples, he cites to Gödel's Incompleteness Theorem (discussed below) and quantum theory. With reference to the latter, Penrose observes that the universe cannot be explained solely in terms of classical, determinate physics. Instead, certain microscopic parts of the universe interact in a non-algorithmic fashion.

Penrose extrapolates from quantum theory and predicts that the human mind also has a “non-algorithmic ingredient.” Although “unconscious actions of the brain . . . proceed according to algorithmic processes, the action of consciousness is quite different, and it proceeds in a way that cannot be described by any algorithm.” Furthermore, “judgement-forming that . . . is the hallmark of consciousness is itself
something that the AI people would have no concept of how to program on a computer."\textsuperscript{123}

We are not currently able to forecast what implications AI might hold for language and the law. Our guess is that should the strong AI advocates be correct about human consciousness, a much better case can be made for objectivity in human language. Given a strong AI vision of the world, duplication of the human mind can be achieved because all thought processes are algorithmic. In this world the brain's process for assigning meaning to words would also be algorithmic. Rationality would thus form the norm, and language would appear capable of projecting a core, objective meaning. If Penrose is correct, on the other hand, then deducing meaning from words and phrases would likely prove to be a non-algorithmic process. This would provide greater credence to speculations about inherent subjectivity in language.

We do not purport to offer here any concrete answers to these delicate linguistic questions. Instead, we only offer a tentative, intuitive observation that the case for linguistic vagueness seems overstated. We do not doubt that language is vague, but we are circumspect over whether it must be. At the least, we seriously question whether it must be vague in a way that mathematics is not. Commentary to date has focused almost entirely on the practical limitations of human language. Though this discussion has proved useful in deconstructing the notion that legal English constrains judicial decisionmaking, it has yet to address convincingly whether such linguistic constraints are inherently subjective and indeterminate. We will therefore proceed under the assumption that legal English can support a formally constructed system. We acknowledge, however, that much work needs to be done to render our assumption a reality.

B. Reaching for Formality Within the Law

Assuming legal English is objective enough to sustain formality, two additional requirements must be met before the law can be considered rigorously formal. First, self-evident truths must be established as legal axioms, and second, procedural rules that guide legal reasoning must be developed.\textsuperscript{124} In short, one must design a formal system implementing the law. In the following sections, we will demonstrate that no such system is possible, even under these assumptions. But for now, let us consider what form such a system might take.

\textsuperscript{123} Id. at 412 (emphasis in original).
\textsuperscript{124} See supra text accompanying notes 31-32.
We envision that a formal model of the law will presumably supply precedent (whether statutory or common-law) for its axioms. Axioms so expressed may not be self-evident, as are those of arithmetic, but would be manifest in a positivistic sense. Legal logic would motivate the rules for deriving new "theorems" from the precedent axiom set.125 We do not deny the possibility of other models, and our subsequent analysis does not require this approach. However, models that diverge substantially from that which we envision here might well be too different to serve as practical models of "the law" that society is willing to accept.

For example, consider Professor Joseph Singer's proposal of a formal system consisting of just one rule: All plaintiffs lose.126 This system, which we shall call \( P\text{-Loses} \), appears complete, for it provides a solution for every legal dispute,127 and consistent, because in no case will it permit a court to deduce both that one party must prevail and then again that this same party must lose. Although this system has been characterized as a "legal system" by some scholars,128 we do not view it as a practical formal model of the law for two reasons.

First, although it provides a single, ostensibly determinate solution for any litigation, \( P\text{-Loses} \) fails to incorporate even the simplest of common-law rules and logic.129 It instead uses a single, simple, and brutish rule to achieve its results. In this way, it resembles an analogous system found in mathematics: the elementary logic of propositions, also known as the calculus of tautologies.130 This system can also be proved consis-

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125. See, e.g., Sinclair, supra note 24, at 363 n.33. Our reference to "legal logic" here is a reference to deductive legal reasoning, as opposed to inductive or analogical legal reasoning. Only deductive reasoning, which proceeds from general rules to specific conclusions, has the power to produce formal results. See Wellman, supra note 89, at 64-87. "Unlike deductive arguments, inductive arguments do not necessitate their conclusions." Id. at 86. Rather, reasoning by analogy is "probabilistic." Id. at 92. Professor Wellman argues in favor of a third form of legal reasoning: "practical reasoning." Id. at 88. He admits, however, that this form of reasoning is not determinate, and we will therefore not consider it here.

126. Singer, supra note 6, at 11.

127. But see infra notes 133-136 and accompanying text.

128. See, e.g., Ken Kress, Legal Indeterminacy, 77 Cal. L. Rev. 283, 286 (1989) ("Although complete determinacy is attainable in a legal system (Singer considers the rule: The plaintiff always loses), any completely determinate system would fail to be 'just or legitimate' because it would insufficiently protect 'security, privacy, reputation, freedom of movement,' and other competing values.") (citing Singer, supra note 6, at 11).

129. By its own terms, any rule of law providing a remedy under some circumstances is imperfectly approximated by \( P\text{-Loses} \) for any case where those circumstances obtain. For any case where a plaintiff might win damages under the common law—for example, a non-consensual and unprovoked battery—\( P\text{-Loses} \) derives an "invalid" result.

130. A tautology is a statement that excludes no logical possibilities. Ernst Nagel & James R. Newman, Gödel's Proof 52 (1958). An example is any statement of the form "\( P \) or not \( P \)." This statement is necessarily true, no matter how the non-logical variable is inter-
tent in an absolute sense, even when the non-logical symbols are arithmetic formulae.\textsuperscript{131} However, the calculus of tautologies only admits as theorems those propositions of arithmetic that are \textit{trivially} true, that is, those that are true regardless of the validity of the underlying arithmetical propositions.\textsuperscript{132} It is thus a severely limited system that can only derive true propositions of arithmetic that are also tautological.

Only in the most picayune sense can the calculus of tautologies be considered a formal model of arithmetic. True, its theorems are all correct propositions about the objects of arithmetic, and the system cannot produce a contradiction. But the calculus of tautologies is of limited value to people who want to uncover even the most elementary truths about numbers.

For like reasons, \textit{P-Loses} models the law in only the most trivial sense imaginable. It offers nothing to help resolve societal disputes—a fact recognized by scholars on both sides of the formalism debate.\textsuperscript{133} A practical formal model of the law must have the strength to provide useful answers. This is the reason one aspires to formality in the first place.

Second, even assuming that a tautological system can be formally useful, \textit{P-Loses} still fails as a counter-example. A close examination of \textit{P-Loses} suggests that it is not even a tautological system. \textit{P-Loses} is complete only if one assumes there can be no dispute over who or what is a "plaintiff." Does "plaintiff" include those who make counterclaims?\textsuperscript{134} What about third-party claims?\textsuperscript{135} Are those who are involuntarily joined plaintiffs?\textsuperscript{136} Answering these questions requires additional rules and a formal methodology. Hence, it appears that \textit{P-Loses} may be relieved of its tautological character after all, and may be transformed into a more interesting (and powerful) system. As established below, it is the very power of an "interesting" system that necessitates incompleteness or inconsistency.\textsuperscript{137}

\textsuperscript{131} \textit{Id.} at 45-56.

\textsuperscript{132} Hofstadter offers additional examples of limited formal systems that involve operations of arithmetic but are complete and consistent. \textit{HOFSTADTER, GOLDEN BRAID, supra} note 26, at 406-07.

\textsuperscript{133} \textit{See}, \textit{e.g.}, \textit{D'Amato, One Bold Thought, supra} note 72, at 113-14 ("I would contend that such a 'system' is not and cannot possibly be a 'legal system.' "); \textit{Kress, supra} note 14, at 286; \textit{Singer, supra} note 6, at 11.

\textsuperscript{134} \textit{See} \textit{FED. R. CIV. P.} 13 (compulsory and permissive counterclaims).

\textsuperscript{135} \textit{See} \textit{FED. R. CIV. P.} 14 (third-party practice).

\textsuperscript{136} \textit{See} \textit{FED. R. CIV. P.} 19(a) (joinder of persons needed for just adjudication).

\textsuperscript{137} \textit{See infra} notes 148-158 and accompanying text.
Assuming that our imagined legal system is formally constructed and determinate, it must mechanically resolve all cases that arise under its terms. For any dispute, whatever the facts, the law will render an outcome without needing human assistance. This outcome would constitute a proposition of the legal system, and would look something like this: “Given facts \( A, B, C, \ldots \), propositions \( q, r, s, \ldots \), therefore \( P \).”\(^{138}\)

Of course, because an infinity of facts (and hence possible cases) exist, the number of derivable legal propositions is unbounded. The law is thus ideal, not only for the reasons mentioned above, but also because it allows a finite, manageable collection of rules to direct all of society's affairs and incursions.

Were the law truly formal and determinate, a computer could, in theory,\(^{139}\) be programmed to apply it.\(^{140}\) Indeed, this computer would act as the law in every way, with its only limitation being an inability to develop facts.\(^{141}\) Facts would instead have to be determined by some human decisionmaker and then fed into the program. Once this factual data is input into the program, however, the program would algorithmically apply the proper rules and render the correct result. Such a program may not be exactly what mainstream legal scholars have in mind,\(^{142}\) but it must follow from a true appraisal of formalism. The de-

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\(^{138}\) \( P \) represents any legal conclusion, including an intermediate conclusion of law, such as “a valid contract was formed,” or an ultimate resolution of the case, such as “the plaintiff wins.”

\(^{139}\) Consider the following outline of such a program. It is a simple matter to generate all possible strings of text that can be produced, given an alphabet: first all strings one character in length, then longer strings, and so forth. Since the law, by hypothesis, is formal, we can algorithmically verify whether or not each string constitutes a dispositive proof of any given legal matter; for in a formal system, the validity of a string is determined algorithmically from the string's syntax alone. The program continues this process of generating the next string and verifying whether it is the dispositive argument for one or the other side. When one is found, it halts.

Since the law is determinate, hence complete, there must exist a legal argument to resolve the matter. Thus, the program must halt eventually. Because the law is consistent, once we find such an argument, we are secure that no dispositive argument to the contrary exists. Thus, the result of the program determinately resolves the matter. See generally Penrose, supra note 30, at 116-18.

Although a program that would determine legal disputes must theoretically exist, given these assumptions, the algorithm need not necessarily be efficient or practical. The number of possible strings, even those of moderate size, is intractably large. For example, the class of all possible text strings of length 40 from an alphabet of only ten characters numbers \( 10^{1027} \). Processing all such strings at the rate of one thousand per second would take more than three octillion (\( 3 \times 10^{32} \)) centuries.

\(^{140}\) See generally D'Amato, Computers, supra note 89; Penrose, supra note 30, at 99-148.

\(^{141}\) See D'Amato, Computers, supra note 89, at 1279-80.

\(^{142}\) See, e.g., Schauer, Formalism, supra note 7, at 536-39 (complaining that formalism's
rivative question of legal formalism, then, is whether a program of this
nature, at least in theory, exists. We intend in the succeeding sections to
show that it does not.

1. Hilbert's Entscheidungsproblem

David Hilbert, a famous German mathematician, posed an identical
question—called the Entscheidungsproblem—about mathematics in
1900. Hilbert was so impressed with the power of mathematics that
he believed all of mathematics must somehow be related, and that a gen-
eral, formal decision process could be found that would solve all math-
ematical questions. Hilbert challenged mathematicians to develop such
a system, or to prove that it did not exist. He sought no less than to
insulate mathematics from human judgment and intuition. Once a gen-
eral decision system was discovered, all insight would cease. Problems
would be input into the system and algorithmically solved without fur-
ther human thought.

Hilbert's proposal, of course, predated the age of electronic com-
puters. Cast in modern terms, Hilbert was seeking something analogous
to the computer program described above. Following the initial input,
the program would solve the problem without asking further questions of
the programmer. It would not give the programmer a "menu" to choose
from, nor would it ask the programmer to select a specific routine. It
would ask nothing at all. It would simply run until it solved the
problem.

2. Gödel's Incompleteness Theorem

Fortunately for mathematicians (lest they be out of work), Kurt
Gödel proved in 1931 that Hilbert's proposed formal system does not
exist. Specifically, Gödel demonstrated that formal systems powerful
enough to express the axioms and propositions of arithmetic cannot be
closed and rigorous nature acts as an impediment to "optimally sensitive decisionmaking"); see also D'Amato, Computers, supra note 89, at 1298-1301 (evaluating the costs of a formal legal
system).

143. Penrose, supra note 30, at 34.
144. Id.
145. See id. at 110-12.
146. See supra notes 139-141 and accompanying text.
147. See Penrose, supra note 30, at 105. Of special note is Penrose's remark:

The point of view that one can dispense with the meaning of mathematical state-
ments, regarding them as nothing but strings of symbols in some formal mathemati-
cal system, is the mathematical standpoint of formalism. Some people like this idea,
whereby mathematics becomes a kind of "meaningless game." It is not an idea that
appeals to me, however. It is indeed "meaning"—not blind algorithmic computa-
both complete and consistent. Instead, these systems must contain either statements that are neither provable nor disprovable within the system, or at least two inconsistent statements that are both provable within the system. In either case, proof or disproof of these statements can only be accomplished with extra-systemic insight.

Gödel's proof was complex, delicate, and closely tied to the specific formal system he considered. Briefly, it followed from recognizing that an arithmetical proposition, $G$, can be properly constructed within arithmetic, where $G$ has the following meaning:

\[ G \]
The arithmetical proposition $G$ has no proof within the system of arithmetic.

Gödel demonstrated that this particular proposition is in fact true, and therefore cannot be proved under the axioms and rules of arithmetic, unless arithmetic is itself inconsistent.\textsuperscript{151}

In simple logic, Gödel's proof proceeds this way: Ask the question whether $G$ has a proof within arithmetic. If it does, then what it asserts about arithmetic must be true. But what $G$ asserts about arithmetic is that $G$ has no proof. Therefore, there is no proof of $G$ in arithmetic, contradicting the original assumption that a proof for $G$ exists. Conversely, inquire whether there is a proof of the negation of $G$, $\sim G$, within arithmetic. If such a proof exists, then what it asserts about the system would also be true of arithmetic, that $\sim G$ has a proof within arithmetic. Consequently, from the assumption that $G$ has a proof, we derive a proof for $G$ and must therefore conclude that arithmetic is inconsistent. Thus, arithmetic either has a proposition which can be neither proved nor disproved, or it is inconsistent.

Gödel's proof demonstrated that Hilbert's program was doomed to failure. Absent extra-systemic insights, such as thought or intuition, no mechanical system can derive all true propositions of arithmetic. From this, mathematicians recognized that to perform higher mathematics, some "informal" metamathematical reasoning is necessary. A computer program may assist mathematicians, but cannot supplant them. Although useful for guiding and validating mathematical work, designedly formal systems such as arithmetic are by their very nature doomed to play a secondary role in the search for truth.\textsuperscript{152}

3. \textit{The Limits of Gödel's Proof}

The true genius of Gödel's proof lies not so much in its results as in Gödel's manner of constructing the proposition $G$. Propositions of formal systems do not naturally have meanings; they are merely abstract objects, strings of the formal syntax of a system's language.\textsuperscript{153} Furthermore, to the extent meaning is attached to propositions, those proposi-


\textsuperscript{152} See Penrose, \textit{supra} note 30, at 112 ("[Formal] systems indeed have very valuable roles to play in mathematical discussions, but they can supply only a partial (or approximate) guide to truth. Real mathematical truth goes beyond mere man-made constructions.").

\textsuperscript{153} Hofstadter, \textit{Golden Braid}, \textit{supra} note 26, at 35. However, meanings develop from the form of the formal system itself. By finding relationships between the form of the theorems of a formal system and some other entity, these relationships, or mapping, induce a meaning onto the strings. \textit{Id.} at 51. For an example, see \textit{infra} notes 155-158 and accompanying text.
tions generally "speak" only in terms of the subject of the formal system and not about the formal system itself. For example, arithmetical propositions naturally "speak" of the properties of numbers, but say nothing about arithmetic itself.154 Hence, it seems that the string $G$ could not really express the meaning Gödel attributed to it.

At least, not at first glance. Gödel observed that based upon the syntax of the strings of the formal arithmetical system, a number could be assigned to each proposition.155 He demonstrated that any proposition—or more accurately, any metaproposition about propositions of arithmetic—can be expressed as a statement about numbers, and hence as a statement within arithmetic.156 This process of associating, or "mapping," strings within a system with propositions about the system is known as "embedding."157 It is this technique, coupled with Gödel's intricate numbering scheme, that allowed Gödel to ultimately demonstrate the true existence of his $G$ proposition.158

Gödel's proof consequently finds itself closely associated and intertwined with arithmetic. Related systems may be proved to suffer arithmetic's fate, but only after they are either seen to contain arithmetic or somehow correspond with arithmetic such that they can be used to prove arithmetical propositions.159 Absent an explicit construction of this type, Gödel's result does not assist the discussion regarding a system's inherent indeterminacy.

When mathematical systems are at issue, this observation about the scope of Gödel's Theorem has proved unimportant. After extensive research and analysis, all proposed mathematical systems were found to be

155. A "Gödel number" is assigned to each character in the alphabet of arithmetic. See, e.g., Gödel, On Formally Undecidable Propositions, supra note 148, at 601 (coding the numeral "0" as the number 1; the symbol for negation "~" as 5; the symbol for disjunction "v" as 7; and etc.); see also Hofstadter, Golden Braid, supra note 26, at 268; Nagel & Newman, supra note 130, at 68-76. The Gödel number for a string of characters is a function of the Gödel numbers of its constituent characters, contrived so that the Gödel number for each string is unique, and so that the Gödel number of its substrings is given by an arithmetical function of the string itself.
156. Nagel & Newman, supra note 130, at 76-84. Essentially, he showed that there was a correspondence between statements about propositions (metapropositions) and statements about the corresponding Gödel numbers for those propositions, such that the metaproposition was true about propositions of arithmetic if, and only if, the corresponding proposition within arithmetic was true of numbers. This process was called the "arithmetization" of metamathematics. Id.
157. Hofstadter, Golden Braid, supra note 26, at 97.
159. See infra Part III.B.4.
sufficiently equivalent to arithmetic that Gödel's Theorem could be applied. Mathematicians since then have simply assumed that all new formalizations would fall subject to the same analysis. Consequently, general statements about Gödel's Theorem prove correct when speaking about mathematics, though even these are to some extent mere conjecture.

Whether the law bears some correlation to arithmetic such that Gödel's Theorem can be applied is a difficult question. Some have relied on intuition and assumed that it does. They argue that if Gödel's Theorem is true of a limited, rigorous system such as arithmetic, a fortiori it must also be true of a richer, more amorphous system such as the law.

Others, however, are more skeptical. Professor Randall Kelso, for example, has argued that the correspondence between statements within arithmetic and statements about arithmetic found in Gödel's proof simply cannot exist with respect to questions of nature, or consequently of the law. Although this argument identifies the difficulty in finding such a correspondence, it fails to demonstrate that such mapping is impossible. Moreover, the assumption underlying this posi-

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160. This assumption, although well founded on the experience of mathematical logicians, has never been proved:

The possibility of constructing a finitistic absolute proof of consistency for arithmetic is not excluded by Gödel's results. Gödel showed that no such proof is possible that can be represented within arithmetic. His argument does not eliminate the possibility of strictly finitistic proofs that cannot be represented within arithmetic. But no one today appears to have a clear idea of what a finitistic proof would be like that is not capable of formulation within arithmetic.

NAGEL & NEWMAN, supra note 130, at 98 n.31 (emphasis in original).

161. See, e.g., D'Amato, Can Legislatures Constrain?, supra note 7, at 598 n.96; D'Amato, Pragmatic Indeterminacy, supra note 9, at 176 n.91; Roy L. Stone-de Montpensier, The Compleat Wrangler, 50 MINN. L. REV. 1001, 1002 (1966); Roy L. Stone-de Montpensier, Logic and Law: The Precedence of Precedents, 51 MINN. L. REV. 655, 664 (1967) (assuming Gödel applies to the common law, thus rendering the common law incomplete); see also KORNSTEIN, supra note 61, at 127 ("The implications of Gödel's Theorem for any theory of the law have been ignored for too long . . .").

162. See, e.g., Sinclair, supra note 24, at 363 n.33 (1987) (challenging Stone-de Montpensier for not explaining "why we should think Gödel's theorem relevant to [the common-law] system.").

163. Kelso, supra note 9, at 830-32.

164. Id. at 832. Kelso observed that there must be a special correspondence between the system and metasystemic propositions, as there was in Gödel's theorem, as well as mere potential for self-reference. He suggested that the existence of such a correspondence in a model of natural law does not follow from Gödel's theorem alone. Id.

165. See id. at 832-35 ("Once reality is assumed to exist . . . Gödel's theorem places no limits on the ability of a system of legal rules or principles to be complete and consistent."). Kelso fails to make any substantive argument why the correspondence between the legal and the metalegal cannot be made. In particular, although he observes correctly the use and mention errors arising out of self-reference in the Epimenides paradox, id. at 829-39; see also infra
tion—hinging on whether reality is illusory—cannot be seriously considered. In any event, Gödel's Theorem can hardly be dismissed as having no application to the "real world."

Professor Ken Kress presents a more modest suggestion. Rather than challenging the meaning of Gödel's Theorem, he questions whether it has any application to the law. Specifically, Professor Kress argues that certain rigorous mathematical proofs are inapplicable to the English language. These proofs, Kress argues, "will not 'go through' in legal English." He states: "'Words,' English, and legal language are insufficiently precise for the assertions and inferences of a formal proof ... to be true and valid about them." Although Professor Kress does not make this charge in specific reference to Gödel's Incompleteness Theorem, his objection is broad enough to encompass the issue of that theorem's applicability to the law.

Part III.B.5 (discussion of Epimenides paradox), he seems unaware of Professor W.V. Quine's restatement of the paradox that avoids this error, see infra text accompanying notes 177-178, even though Quine's work appears prominently throughout the book reviewed. See, e.g., Hofstadter, Golden Braid, supra note 26, at 431-37, 445, 446, 449, 497-99, 531, 698-99. Thus, Kelso's arguments merely beg the question whether Gödel's Theorem can apply to the law. While Kelso claims that Gödel's Theorem is meaningless, given "the assumption of reality," see Kelso, supra note 9, at 832-34, other mathematicians and philosophers disagree. Israel Kleiner, Rigor and Proof in Mathematics: A Historical Perspective, 64 MATHEMATICS MAG. 291, 307 (1991) ("Gödel's results are of fundamental philosophical consequence ... "); see S.G. Shanker, Wittgenstein's Remarks on the Significance of Gödel's Theorem, in GÖDEL'S THEOREM IN Focus, supra note 23, at 155, 241-42. Roger Penrose, for example, envisions Gödel's Theorem as an awakening of true consciousness:

Mathematical truth is not something that we ascertain merely by use of an algorithm. I believe, also, that our consciousness is a crucial ingredient in our comprehension of mathematical truth. We must "see" the truth of a mathematical argument to be convinced of its validity. This "seeing" is the very essence of consciousness. It must be present whenever we directly perceive mathematical truth. When we convince ourselves of the validity of Gödel's theorem we not only "see" it, but by so doing we reveal the very non-algorithmic nature of the "seeing" process itself.

Penrose, supra note 30, at 418 (emphases in original).

Kress, supra note 14, at 144.

Id. Professor Kress makes this assertion specifically about the Löwenheim-Skolem theorems. See infra note 168 and accompanying text.

Professor D'Amato (whose article Kress was challenging in the latter's assertion about mathematical proofs) assumed in a later rebuttal that Kress was in fact speaking to Gödel's Theorem. D'Amato replied:

Although Professor Kress is technically correct in saying that Löwenheim-Skolem (as well as Gödel-Church) were designed to apply to formal systems, my position is that either they apply a fortiori to non-formal systems such as law, or if they don't apply because law is a non-formal system, then for that reason the Indeterminacy thesis is proven.

D'Amato, Pragmatic Indeterminacy, supra note 9, at 176 n.92.

Professor D'Amato's apparent horns-of-a-dilemma arguments neglect the possibility that there might be deterministic methods other than those which use formal systems. In mathe-
Given the delicacy of Gödel's proof, Professor Kress's suggestion that it may not "go through" in legal English is certainly credible.\textsuperscript{169} Legal English could prove objective enough to sustain a formal structure while avoiding altogether the limiting implications of Gödel's Theorem. On the other hand, the expressiveness of arithmetic—not its limitations—allowed Gödel to demonstrate its indeterminacy. Because legal English is more expressive than arithmetic, the intuitive argument that Gödel's results must apply equally to the law has a certain appeal. Though intuition is by no means a convincing proof,\textsuperscript{170} it prompts us to explore whether one exists.\textsuperscript{171}

4. Applying Gödel's Theorem to the Law

We envision four methods for demonstrating the application of Gödel's Theorem to legal English. First, a direct proof along the lines of Gödel's original proof might be constructed. Given, however, the complexity of Gödel's proof and the density of any formal model of the law,

\begin{itemize}
  \item matics, for example, it is demonstrable that there is no algorithm to determine whether a given computer program will halt. \textit{Penrose, supra} note 30, at 57-65 (proving Turing's halting theorem). Nevertheless, it is certain that all computer programs will either halt eventually, or they will not. Thus, despite the absence of a formal means to determine the result, it may be meaningful to say that a determinate answer exists. These questions raised by the Kress-D'Amato debate are analogous to the debate between mathematical platonists and mathematical intuitionists. Mathematical intuitionists refuse to accept as true all statements of the form "\(P \text{ or not } P\)," since for some statements, such as the Gödel proposition "\(G\)," neither \(G\) nor its negation can be proved. Platonists hold that even if \(G\) can be neither proved nor disproved, the proposition "\(G \text{ or not } G\)" is necessarily true. \textit{Id.} at 112-16.

\item \textsuperscript{169} \textit{See supra} text at note 166. Still, the manner in which Professor Kress phrases his objection is troubling. He protests the use of mathematical principles because legal language is "imprecise." Kress, \textit{supra} note 14, at 144. As Professor D'Amato explains in response to Professor Kress: "If English is not precise, then Formalism is out the window and we don't need Gödel . . . ; only Indeterminacy remains." D'Amato, \textit{Pragmatic Indeterminacy, supra} note 9, at 172 n.78.

\item \textsuperscript{170} Douglas Hofstadter has warned against rote extension of Gödel's Theorem to other disciplines:

  If one uses Gödel's Theorem as a metaphor, as a source of inspiration, rather than trying to translate it literally into the language of . . . any other discipline, then perhaps it can suggest new truths in . . . other areas. But it is quite unjustifiable to translate it directly into a statement of another discipline and take that as equally valid. \textit{Hofstadter, Golden Braid, supra} note 26, at 696.

\item \textsuperscript{171} Compare the comments of Roger Penrose:

  If we can see that the role of consciousness is non-algorithmic when forming \textit{mathematical} judgements, where calculation and rigorous proof constitute such an important factor, then surely we may be persuaded that such a non-algorithmic ingredient could be crucial also for the role of consciousness in more general (non-mathematical) circumstances. \textit{Penrose, supra} note 30, at 416 (emphasis in original).
\end{itemize}
such an approach appears intractable. Arithmetic is a very simple system, and Gödel’s work in that context was an astounding feat. A similar proof using a more complex system such as the law would require a herculean effort.\textsuperscript{172}

Second, arithmetic might be directly embedded within the law. This would entail showing that propositions of arithmetic can properly be proved using legal reasoning in such a fashion that a given proposition of arithmetic is a theorem of arithmetic if, and only if, it can be proved under the law. If this were done, the law would necessarily contain at least one indeterminate proposition. Were this somehow possible, the indeterminacy of propositions within the law would follow directly from Gödel’s Theorem, since all theorems of arithmetic would also be dispositive propositions of law.

Third, arithmetic might be indirectly embedded within the law. Instead of directly expressing arithmetical propositions as legal propositions, a correspondence might be found between arithmetical propositions and hypothetical cases within the law. The correspondence could be designed, for example, so that if a proposition of arithmetic corresponds to a given hypothetical, then the proposition is a theorem if, and only if, the hypothetical must be determinately decided for the plaintiff. Because Gödel’s proof demonstrates that there are true arithmetical propositions which cannot be proved, all of their corresponding hypothetical cases will be indeterminate.\textsuperscript{173} The case, because of the algorithm’s construction, would be decidable only if the corresponding arithmetical proposition is provable in arithmetic. As in the second approach, the indeterminacy of arithmetic would then prove a general indeterminacy in the law. Not all propositions about arithmetic can be proven within arithmetic; hence, certain cases would not be determinate either.

\textsuperscript{172} Gödel proved his theorem for the simple formalization described in \textit{Principia Mathematica}, see Gödel, \textit{On Formally Undecidable Propositions}, supra note 148, which used but a dozen symbols and about that many proof rules, see \textit{Whitehead & Russell}, supra note 148. Any hypothetical axiomatization of the law will likely be substantially more complex, even in skeletal form. See, e.g., Sinclair, supra note 24 (sketching a formal system of law).

\textsuperscript{173} Assume the converse is true—that is, the law is determinate. If this is so and one could show that all theorems of arithmetic are provable within the law, then the law could be used to construct a general decision procedure (a Hilbert program) for arithmetic. Because the law is assumedly determinate, a general decision procedure exists for determining the outcome of any hypothetical. Resolution of certain cases would prove that certain arithmetical propositions are provable in arithmetic. Indeed, all propositions of arithmetic could be proved in this way. But we know this is not possible because of Gödel’s theorem. Therefore, the assumption that the law is determinate must be false.
Proofs such as these are certainly possible, but hardly obvious. The tediousness and complexity of the first approach, even if it could be accomplished, would prove overbearing. The second and third models, meanwhile, require a very clever construction of a correspondence between legal English and arithmetic. Developing such a construction would, to say the least, be an imaginative task.

Furthermore, before such a proof could even be attempted, one would need to begin with a working formal model of the law—a task that has eluded scholars to date. Each of the first three approaches derives its result from the actual form and structure of a proposed system. Without an exemplar of a legal system, no direct proof using these techniques is possible. Even if one were able to provide a rough outline of such a model—one powerful enough to facilitate a direct proof—no assurance would exist that this same proof could be applied to any other hypothetical model of the law. Although Gödel’s proof has been confidently extended to other models of arithmetic and many branches of mathematics, a similar extension to diverse models of the law is not so certain. Direct Gödel-like proofs might therefore be frustrating in their inability to singularly resolve the debate.

A fourth approach to the problem, that which we take here, is to demonstrate Gödel’s applicability to the law by constructing an indeterminate legal proposition within a specific legal context. Though short of a full-fledged proof, this microcosmic approach, we believe, leads to the same conclusions established by a direct, technical demonstration. Our methodology dispenses with Gödel’s brilliant (and intricate) embedding technique and the necessity of developing an express exemplar of the law.174 We instead opt for a simple—though by no means obvious—illustration. This illustration proceeds along lines similar to the famous Epimenides Paradox.

5. Epimenides and the Law

Linguistic paradoxes premised on the presumption that the truth of a statement depends upon its textual representation date back to ancient times.175 A statement such as “This statement is false” creates a paradox

174. Because this approach does not derive its results from the details of the form and structure of a system, we do not require a functional model of the law. Instead, we merely assume two properties of the system’s behavior: (1) that it “properly” applies one specific rule of law in all cases for which that rule is applicable (duties under a valid contract are enforceable when all conditions precedent are satisfied); and (2) that it does not require a judge to hold true any logical proposition that can be demonstrated false under a given formal system of logic.

175. For instance, the famous Epimenides paradox involved Epimenides, a Cretan, who
when its own truth is questioned. One would assume that either "This statement is false" is true, or "This statement is false" is false. The paradox exists because assuming either of these to be correct admits a line of reasoning proving the validity of the other. The result is an antinomy. Although the propositions are negations of each other, they are both either true or false.

The difficulty in this paradox can be resolved by inquiring as to what exactly the "This" in "This statement is false" refers. Ultimately, one will find that the paradox arises from taking "This" to mean both the sentence—the syntactic object "This statement is false"—and the proposition—the logical object to which the sentence refers. When the two meanings are bifurcated, one need not abandon a usual understanding of the truth predicate. Instead, one need only abandon the idea that a logical proposition must identify with the sentence that expresses it. Once this is done, the sentence "This statement is false" empties itself of any real content.\(^{176}\)

Paradox, however, cannot be so readily disposed of in natural language. Owing to a clever construction by Professor W.V. Quine, the paradox of a textually-based truth predicate can be renewed in a meaningful way. Professor Quine restated "This statement is false" as:

"Yields a falsehood when appended to its own quotation" yields a falsehood when appended to its own quotation.\(^{177}\)

Quine's restatement avoids ambiguous words such as "This," but still accomplishes the self-reference necessary to achieve an antinomy.

To understand why this is so, consider the following statement:

(I) "Contains ten words" yields a falsehood when appended to its own quotation.

The truth of this statement depends on whether the phrase in quotes, "Contains ten words," when appended to an unquoted version of itself, produces a sentence referring to a logically true proposition. The proposition that results from the operation prescribed by (I) is this:

(II) "Contains ten words" contains ten words.

Because the quoted phrase "Contains ten words" actually contains only three words, the proposition asserted by sentence (II) is false, and the stated: "All Cretans are liars." On the assumption that to be a liar means that every statement is false, Epimenides' proposition is a paradoxical self-reference. See Kelso, supra note 9, at 828-30.

176. See Hofstadter, Golden Braid, supra note 26, at 495-99; Kelso, supra note 9, at 828.

proposition asserted in (I) is true. Note, however, that if the quoted phrase read “Contains three words,” then the resulting statement,

“Contains three words” contains three words

would be true. The corresponding variant of sentence (I) would therefore be false.

Return to Quine's restatement of “This statement is false.” If the syntactic operation demanded in that statement is performed, the result is:

“Yields a falsehood when appended to its own quotation” yields a falsehood when appended to its own quotation.

But this sentence is textually identical to Quine's original restatement. Thus, when one queries whether the proposition represented by Quine's restatement is true, a vicious self-reference is created that is no less paradoxical than “This statement is false.” Quine succeeds in recreating the paradox found in “This statement is false,” but without the confusion over use and mention found with the word “This.”

Quine's construction, together with the assumption that some predicate exists for determining the validity of a sentence's meaning, creates an antinomy. The culprit, of course, is the assumption that a general truth predicate exists. In particular, the notion that a predicate can determine the truth of any proposition solely from a textual representation of it within the language itself is incoherent. The truth predicate, so defined, is indeterminate. As a practical matter, the antinomy can be resolved by restricting the scope of the predicate's use. Nevertheless, this restriction does not solve the indeterminacy; it merely shifts the nature of the indeterminacy from inconsistency to incompleteness.

From Quine's antinomy one begins to see that English suffers problems of self-reference quite similar to those arising from Gödel's numberings of arithmetic. Indeed, the similarity between the two is apparent even without a construction relating arithmetic to legal English. This revelation leads us to consider the possibility that a legal case can be assembled which, with respect to legal determinism, is analogous to Quine's version of the Epimenides Paradox. Such a case would prove

178. Id. at 82-84.

179. Since inherent indeterminacies are somewhat counterintuitive, there is a natural tendency to reject them—particularly at first. This makes it somewhat unfair (as well as incorrect) to casually assert that indeterminacies in one instance a fortiori imply another.

Not only lawyers but most people harbor superstitions about the nature of language and meaning. These superstitions assume that meanings reside “in” language somewhat the way furniture resides in rooms—securely “there” where the interpreter can see, identify, and grasp them the way we can see, identify, and grasp tables and chairs.
that indeterminacy is inherent in the law on a level analogous to, if not exactly equal to, that found in arithmetic.\textsuperscript{180}

6. An Inherently Indeterminate Proposition of Law

Consider the following scenario: Professor Langdelle (a very formalistic professor of law) contracts with one of his Critical students, Kurt Gurdelle, to resolve the question of indeterminacy in the law. The contractual terms are as follows: Langdelle promises to pay Gurdelle one hundred dollars when Gurdelle produces a hypothetical case containing a legal proposition for which the law does not uniquely compel an outcome. Gurdelle accepts the terms of Langdelle's offer and busily goes about constructing such an indeterminate case.

Now assume that Gurdelle, believing he has found such a case, delivers a textual description of it to Langdelle and asks for the one hundred dollars. Langdelle, thinking the proof preposterous, refuses to pay. Of course, Gurdelle has a potential action on a contract against Langdelle. The crucial question is whether Gurdelle has performed under the terms of the agreement, a question that can only be answered by directly inquiring whether Gurdelle's proposed case is indeterminate. Specifically, the question is whether the law compels a unique outcome for the hypothetical situation proposed by Gurdelle.

With this in mind, we need to focus on Gurdelle's hypothetical. Gurdelle realizes that he can succeed in creating an indeterminate fact pattern if he can somehow generate an antinomy like that achieved by Quine. To do so, he observes that he must somehow make his hypothetical refer back to itself. In this way, the hypothetical will take on the character of Quine's restatement of "This statement is false"—thereby becoming indeterminate.

To achieve this result, Gurdelle develops the following situation: He assumes a hypothetical, formalistic law professor contracts with a hypothetical student to prove indeterminacy in the law. The professor promises to pay the student one hundred dollars if the student can

\textsuperscript{180} The petard upon which Gödel ultimately hoisted arithmetic was the ingenious coding of assertions about arithmetic as assertions about numbers. \textit{See supra} notes 155-158 and accompanying text. Here, Quine's result suggests that we may be able to demonstrate, using his technique, the impossibility of a language-based predicate that derives the truth of certain legal propositions. The only difficulty in such an approach is to avoid confusing the use and mention of a legal proposition.
demonstrate an indeterminate case containing a legal proposition that cannot be resolved under the law. The student busily goes about constructing the example and then presents it to the professor. The professor rejects the proof as preposterous and refuses to pay.

Gurdelle styles his not-so-hypothetical case Critical Student v. Formalist Professor. Because the hypothetical case is identical to Gurdelle's case, the judge's ruling on the hypothetical will control the outcome of the real case. Consequently, depending on the hypothetical student's construction of his hypothetical, a vicious self-reference may be achieved.

Suppose now that the hypothetical plaintiff, Critical Student, proposes the following as his proposition that cannot be determined by the law:

The law does not compel a determination that this proposition is true when it is offered as an instance of an indeterminate proposition of the law.

Consider the judge's dilemma when deciding how (and indeed whether) the law determines the validity of this proposition. She might simply assume that the law directs its truth or falsity, and then analyze both possibilities to ascertain why this is so. In regard to the former, if the law directs that the proposition is true, then what it asserts about the law must be true—the law, by the terms of the proposition, does not compel a determination that the proposition is true. Consequently, the judge is compelled to hold that the proposition is true when, in fact, the law does not compel a finding that the proposition is true. This is a contradiction.

Conversely, suppose the judge were to hold that the law compels that the proposition is false. Then, from the above analysis, the negation of what the proposition asserts about the law is true. Because the proposition asserts that the law does not compel a finding that it is true, its negation, that the law compels a finding that it is true, must be true. Thus, the judge is compelled to hold that the proposition is false when, in fact, the law compels a finding that the proposition is true. Again, this is a contradiction.

Because alternatively assuming that the law compels holding the proposition is true and then false yields contradictions, an error must arise from the initial assumption that the law is determinate and directs the validity of the proposition. This does not mean that the proposition itself is neither true nor false in some ideal sense, but merely that the law, in respect to this particular proposition, is indeterminate.

Unfortunately, the analysis conducted above suffers from precisely the same confusion of use and mention that occurred with the sentence
"This statement is false." The indeterminacy arises from an abuse of language and not from the notion of legal determinacy itself. But just as the mistake of use and mention was purged from "This statement is false," so too can it be exorcised from the proposition "The law does not compel a determination that this proposition is true."

Consider the following variant of the latter proposition:

"yields a statement for which the law does not compel a finding that it is true, when appended to its quotation" yields a statement for which the law does not compel a finding that it is true, when appended to its quotation.\(^\text{181}\)

Analyzing this proposition results in precisely the same conclusion: The law is necessarily indeterminate.\(^\text{182}\) Assuming the existence of a complete system that can consistently resolve all legal propositions that can be described in legal English results in an antimony. Using Quine’s construction, that contradiction can no longer be understood as a mere abuse of language. Hence, just as Quine showed that the idea of a text-based predicate resolving truth is logically incoherent,\(^\text{183}\) so too is the notion of a determinate, formal legal system.

181. We have taken some liberties with this version of the proposition for simplicity and clarity. A more rigorous example might be:

"yields a statement for which, when presented to a court as an example of an indeterminate proposition of the law under circumstances where the question of its determinacy is necessary to resolve the dispute in question, the law does not compel a court to determine that it is true, when appended to its own quotation" yields a statement for which, when presented to a court as an example of an indeterminate proposition of the law under circumstances where the question of its determinacy is necessary to resolve the dispute in question, the law does not compel a court to determine that it is true, when appended to its own quotation.

182. Assume, arguendo, that the law compels a unique determination of the truth of the proposition. The law then must compel either a finding that the proposition is true or a finding that it is false. If the law compels a finding that the proposition is true, then what it asserts about the law must be true. Its assertion about the law is that the law does not compel a finding that the proposition resulting from appending "yields a statement for which the law does not compel a finding that it is true, when appended to its quotation," to its quotation, is true. But the result of that operation is textually identical to the text of the proposition that we assumed the law compelled to be true. This is a contradiction.

On the other hand, if the law compels a finding that the proposition is false, then its assertion about the law must be false and the negation of that assertion true. The proposition must therefore mean that the law compels finding the proposition, the text of which results from appending "yields a statement for which the law does not compel a finding that it is true, when appended to its quotation" to its quotation, to be true. But the result of that operation is textually identical to the text of the proposition that we assumed the law compelled to be false. Again, this is a contradiction.

Because in each case a contradiction results, the assumption that the law is determinate must be false. Consequently, the law must be indeterminate with respect to this proposition.

183. See supra Part III.B.5.
Granted, the failure of the law to determine this case can be "repaired" by eliminating the case from the legal system. Such an adjustment, however, would effectively admit fragmentation in the law and hence indeterminacy. Our conclusion is that the law cannot be modeled in a determinate fashion. An indeterminate proposition can be revealed for any interesting construction of the law.

Of course, we have not proved that every model of the law must be inconsistent or incomplete. We have shown, however, that any system roughly analogous to the common law of contracts, which enforces agreements when conditions precedent are satisfied, and which does not require that propositions that are logically false be held true, is demonstrably indeterminate. Any legal system that rejects such fundamental properties might be determinate but, like \( P \)-Loses, would be suspect due to its trivial nature.\(^{184}\) We conclude that the full implications of Gödel's Theorem must apply to any meaningful formal model of the law.

IV. Legal Implications of Gödel's Theorem

A. Hard and Easy Cases

The main question that emerges is "What does all of this mean for lawyers and judges?" The "easy" answer is that the law (as we have imagined it) cannot be a determinate, formal decision process. It cannot mechanically solve all disputes. Instead, human insight and intuition must play a part. Just as creative mathematicians are needed to help us solve the riddles of the universe, intelligent lawyers and judges are needed to resolve the more demanding of society's problems.

But what of the specific ramifications of Gödel's Theorem? Are only contrived cases that refer to themselves affected,\(^{185}\) or do the implications of Gödel's result run deeper? Are certain cases determinate notwithstanding Gödel's Theorem, such that human participation is not required in relation to these particular cases? If not exactly determinate, are certain cases "easier" than others? Is the use of human intuition so minimal in certain cases that we can be confident of the result?

\(^{184}\) See supra notes 126-137 and accompanying text.

\(^{185}\) Self-reference in the law is more common than one might think. See Farago, Intractable Cases, supra note 4, at 205-07 (discussing self-reference in the law); see also Banner, supra note 24, at 244-49 (noting several self-references in the law). Appellate judges sometimes directly engage in Epimenides-like reasoning while determining cases. See generally John M. Rogers, "I Vote This Way Because I'm Wrong": The Supreme Court Justice as Epimenides, 79 Ky. L.J. 439, 442-58 (1991) (noting "150 Supreme Court cases that might have resulted in a justice's decision to vote against his own analysis").
To answer the first of these questions, we note that the impact of Gödel's result extends far beyond self-referential cases such as *Gurdelle v. Langdelle*. Gödel's Theorem runs deeper, reaching every case that can be imagined. For in any case whatsoever, human judgment must play a factor in answering questions about the law. Recall the computer program envisioned in Part III. Gödel's result demonstrates that such a general decision program is impossible. Following the input of factual data, human judgment must be referenced to enable the program to solve the problem. At a minimum, a programmer must direct the computer to the appropriate algorithm.

Recognizing that the *fact* of human participation cannot be avoided in any case, one might question the *degree* of non-algorithmic insight being used. Specifically, one might argue that the human judgment used in deciding a particular case is minimal in comparison with the formal logic used to decide the case. Or, one might assert that the human choice in one case is not as "difficult" as that in another. The purpose behind either argument, of course, is to minimize the importance of certain human choices, resulting in a hierarchy of cases. Certain choices are minor when compared with the algorithm used to solve the problem, and these cases are relatively "easy." Other choices are "harder," meaning that the corresponding cases are more difficult to resolve.

One who seeks to diminish the importance of human choice must understand that measuring the value of insight is not an algorithmic task. Because value in this regard cannot be algorithmically quantified, comparing the relative complexity of cases is objectively impossi-

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186. *See supra* notes 139-142 and accompanying text.

187. If law is incomplete, then the computer program described, *see supra* note 139, will not terminate for all strings. If an input is an indeterminate case, the program will never find a dispositive argument, and will simply go on executing forever. Moreover, although determinate cases will still produce correct results, a hypothetical judge, waiting for the output of a law machine, would not be able to tell if the machine would ever produce a result unless it had already done so.

What is worse, if law is inconsistent, the program—even if it does terminate—does not assure that an argument for a contrary position is impossible. Thus, to assure that the case is not "hard," the judge must wait to see if it produces a contrary argument. However, if the case does in fact have a determinate answer, then the machine will never produce a contrary argument. Either way, the judge must wait forever before getting a certain result from the machine, even for those "easy" cases that are not indeterminate. *See PENROSE, supra* note 30, at 117-18.

188. *See id.* at 63-64; *cf.* Sartorius, *supra* note 65, at 1269 ("There is a uniquely correct result in the vast majority of cases, and there is no reliable judicial criterion for identifying those hard cases in which there is not.").

189. *See PENROSE, supra* note 30, at 412.
We can never know in a formal way whether the insight that directs a person to choose A over B is more or less complicated than that which steers the same person to choose Y over Z. So long as choices exist, establishing a hierarchy of cases or arguments from easy to hard is itself a matter of insight and choice. It follows that identifying "good" arguments, or a range of acceptable arguments, is a non-algorithmic task.

Take the infamous case of the under-aged president. The Constitution expressly provides that the president must be at least thirty-five years of age. Consequently, the argument goes, a twenty-nine-year-old cannot be elected president. Any case involving a twenty-nine-year-old candidate seeking the presidency would thus be easily resolved. All the judge has to do is apply the prohibition of Article II of the Constitution. The only human insight involved, if any, would lie in choosing to look at this specific prohibition—a relatively simple choice.

Article II, however, does not provide the only choice. Professor D'Amato has aptly observed that several arguments can be made in favor of the twenty-nine-year-old candidate. An argument can be made that no one has standing to challenge the candidate's age. Even if a court were to reach the merits, it might be argued that the intent of the framers was to exclude only immature candidates. Better yet, one might argue that the Fifth and Fourteenth Amendments prohibit age discrimination, and that the provision in Article II to the contrary has thus been amended.

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190. Penrose explains this further:
For a simple example, one will have learnt the algorithmic rules for multiplying two numbers together and also for dividing one number by another (or one may prefer to engage the assistance of an algorithmic pocket calculator), but how does one know whether, for the problem in hand, one should have multiplied or divided the numbers? For that, one needs to think, and make a conscious judgment. Id. at 413 (emphasis in original).

191. Professors Rogers and Molzon are consequently incorrect when they argue that an interpretive technique within the law may render "millions of interpretations" impermissible. See supra text accompanying note 20.

192. See Schauer, Easy Cases, supra note 71, at 402, 414. Judge Frank Easterbrook was the first to posit this example. See Frank H. Easterbrook, Statutes' Domains, 50 U. CHI. L. REV. 533, 536 (1983).

193. U.S. CONST. art. II. § 1, cl. 5.


195. Id. at 253 (citing Frothingham v. Mellon, 262 U.S. 447 (1923)).

196. Id. at 251-52.

197. Id. at 255.
To resolve the case of the under-aged president a judge must choose which argument she finds more persuasive. Although formal logic might provide guidance, it cannot resolve the case. Instead, insight ultimately must direct the judge’s decision. And because this insight is not formally measurable, one can only subjectively guess at whether the case is easy or hard.

B. Sequences and Better Solutions

An alternative way to demonstrate indeterminacy in the law (and a resulting absence of easy cases) involves the use of numerical sequences. Professors Tushnet and D’Amato have both observed that given a sequence of three integers—say “2, 4, 6”—no single formula can be deduced that will predict the fourth integer. Put another way, the successor integer to the sequence “2, 4, 6” is indeterminate. Given this mathematical conclusion, the argument goes, it follows that the law too must be indeterminate; if no single formula can be deduced from a sequence of integers, then no single formula can be deduced from a sequence of legal propositions. Moreover, legal English is more expressive and less definite than the set of integers. Hence, legal English presents an even greater opportunity for interpreting a given sequence than does the set of integers.

Undoubtedly, Professors Tushnet’s and D’Amato’s observations about numerical sequences are correct. Basic computation theory proves their point. Given any finite prefix of integers, an infinite number of algorithms exist that generate the same prefix. Thus, given any finite sequence, an infinite number of recipes exist for establishing the next successive number. The sequence “2, 4, 6” can thus be interpreted to mean that the next integer is “8,” or it can be interpreted to mean the

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198. D’Amato, Can Legislatures Constrain?, supra note 7, at 597 n.96; Mark V. Tushnet, Following the Rules Laid Down: A Critique of Interpretivism and Neutral Principles, 96 HARV. L. REV. 781, 822 (1983) [hereinafter Tushnet, Following the Rules]. Professor D’Amato describes this result as follows:

Here is how any prior theory can be used to “explain” any desired result. Suppose we have the following sequence of numbers: 2, 4, 6. I ask you to give me the next number in the series. If you say “8,” you have implicitly applied one possible theory about the sequence. But the Lowenheim-Skolem proofs showed, among other things, that any answer at all to my question can be shown to be the consequence of some theory. Suppose instead you give me the number 10. I then supply an appropriate theory: “the next number in the sequence is derived from adding the two preceding numbers, but when there is only one preceding number, add it twice.” Following this formula, $2 + 2 = 4; 2 + 4 = 6; 4 + 6 = 10$, Q.E.D.

D’Amato, Can Legislatures Constrain?, supra note 7, at 597 n.96 (emphasis in original).

199. See D’Amato, Pragmatic Indeterminacy, supra note 9, at 176 nn.91-92.
successor should be "10." Equally plausible interpretations exist for "1,000,001" and "Jesse Jackson." All of these are "correct."

A problem arises, however, when one attempts to translate this result into legal English. Professor D'Amato attributes the proof of sequential indeterminacy to mathematicians Leopold Löwenheim and Thoralf Skolem. Professor Ken Kress, in response, correctly observes that the proofs of Löwenheim and Skolem hold only for first-order formal systems. Unlike Gödel's proof, which requires a fairly powerful language, the proofs of Löwenheim and Skolem only apply to systems whose base language cannot describe properties. English is a second-order system; it can expressly describe properties of objects (sets) as well as propositions about objects. The proofs of Löwenheim and Skolem therefore do not necessarily hold for legal English.

The natural implication to be drawn from Professor Kress's argument is that legal English is not restricted in the same way as the set of integers. Although a single algorithmic recipe for the sequence "2, 4, 6" cannot be identified, a series of cases can be interpreted to produce a specific, determinate rule. This conclusion, however, is unwarranted for at least two reasons. First, the fact that a mathematical proof does not "go through" into English says nothing about English's limitations. Limitations may still exist; indeed, these limitations might be identical to those disclosed by the mathematical proof. Mathematics simply fails to reveal them.

200. See D'Amato, Can Legislatures Constrain?, supra note 7, at 597 n.96.
201. Id.; D'Amato, Consequences of Plain Meaning, supra note 80, at 572 & n.135. Professor Tushnet, on the other hand, does not cite to these proofs. See Tushnet, Following the Rules, supra note 198, at 822. See generally W.V. Quine, Methods of Logic 209-12 (4th ed. 1982) (surveying the Löwenheim-Skolem Theorems).
202. See Kress, supra note 14, at 144-45. A first-order formal system is one where the variables of the language can only represent individual objects. In second-order systems, the variables of a language can represent properties, or sets, of individual objects. The Löwenheim-Skolem Theorems do not apply to higher-order systems. George Boolos & Richard Jeffrey, Computability and Logic 198-200 (1974).
203. Consider, for example, all collections of sentences describing properties of objects. If English were first-order, we would only be able to extensionally discuss the sentences themselves, as opposed to collections of such sentences. English could only be a first-order language if sentences such as the first sentence in this footnote were meaningless.
204. See Kress, supra note 14, at 144 ("There is therefore no reason to suppose that the conclusion of the proof, the Löwenheim-Skolem theorem, is true in legal English (or "words"). Moreover, as we shall see shortly, there is good reason to think it is false.") (footnote omitted). Whereas Gödel's proof derives the limitations of arithmetic by exploiting the strengths of its underlying language, the Löwenheim-Skolem result arises from the weaknesses of its underlying language. Any language capable of extensionally defining a set admits an express counterexample for the Löwenheim-Skolem theorem. Boolos & Jeffrey, supra note 202, at 147-56.
Second, and more important here, the Löwenheim and Skolem proofs need not "go through" for the principle underlying sequential indeterminacy to apply to legal English. Indeed, the Löwenheim-Skolem Theorems are not even necessary to demonstrate that a sequence of integers—like "2, 4, 6"—cannot be interpreted to produce a single, determinate successor. Basic computation theory is sufficient for this task.

The Löwenheim-Skolem Theorems are thus superfluous to the debate over indeterminacy in the law because they are unnecessary. Any discourse over the analogies between the law and indeterminacy in numerical sequences should focus on common sense rather than the intricacies of Löwenheim's and Skolem's proofs. Moreover, given the application of Gödel's Theorem to legal English, any analysis based on sequential indeterminacy should at best be understood as a schematic illustration.

Gödel's proof supplies the requisite theoretical support because it demonstrates that human insight is always necessary to resolve legal disputes. We have already seen that the degree of human insight necessary for the resolution of any given dispute cannot be algorithmically measured. Hence, for any given case—no matter the precedent, no matter how obvious the sequence—an unmeasurable intuitive choice will factor into the decision process. A sequence of cases analogous to "2, 4, 6" could just as easily be interpreted to mean that the next case must be "125" as it could be read to mean that the next case must be "8". And

205. The Löwenheim and Skolem Theorems establish that a first-order formal system having one model must also have an additional model in the space of countable integers. This does not mean that all sequences, like "2, 4, 6," must admit equally plausible results, like "8" or "10." Indeed, once a model for interpretation is chosen for that sequence there may well be only one result. If anything, the Löwenheim-Skolem Theorems merely establish that more than one model exists for interpreting any given sequence.

206. Given any fixed prefix of integers, there exist an infinite number of algorithms that will satisfy that prefix. Michael Machtey & Paul Young, An Introduction to the General Theory of Algorithms 113 (1978) (For every general recursive function, there exist a countable infinity of functions that share the same input-output properties.). If we are willing to accept that an algorithm somehow embodies a reasonable recipe for determining a sequence, this would suffice to demonstrate the proposition.

Another way to see why this is so is to imagine that there exists a finite prefix for which only one sequence provides a plausible description. Then consider the shortest such sequence which admits only one continuation, and assume that the unique next element of that sequence is x. Now consider the sequence that begins with the shortest sequence admitting only one continuation, and is followed by an infinity of x + 1's. If one accepts that the account just given is a plausible description of a sequence, this contradicts the possibility that the originally hypothesized sequence had only one determinate continuation. This denies the possibility of a plausibly determinate finite sequence prefix.

207. See supra Part IV.A.
neither of these results can be objectively understood as better than the other. From an algorithmic, qualitative standpoint, they are identical.

None of this precludes the existence of a credible, non-algorithmic method for judging the quality of responses and interpretations. As an intuitive matter, some arguments do seem better than others. Eight seems a more probable successor to the sequence “2, 4, 6” than does “125.” “No” seems the better response to the issue of whether a twenty-nine-year-old can be president. But why is this true? One explanation might be socialization. Eight is perceived as the better answer because it is the solution society more commonly recognizes. Another related explanation might focus on simple experience. A person who encounters the same problem again and again soon learns what works and what does not. Although the initial choice is intuitive, experience allows the choice to be reapplied in an algorithmic way.

One of the more intriguing explanations under study today is grounded in natural selection. Certain intuitive solutions are better than others simply because they allow the decisionmaker employing them to survive and prosper. Biological evolution allows certain insights to survive, while others perish with those who employ them. Although “8” is not inherently superior to “125” as a successor to the sequence “2, 4, 6,” it is a better solution because one who would normally prefer “8” is more likely to survive. Moreover, natural selection might allow for the evolution of algorithms that employ their own variety of conscious judgment—these algorithms (heuristics really) might be capable of intuitively judging the validity of other algorithms.

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208. See Tushnet, Following the Rules, supra note 198, at 823 (“we know something about the rule to follow only because we are familiar with the social practices”).

209. Penrose describes the role of experience thus:

Of course, once one has done a large number of similar problems, the decision . . . may become second nature and can be carried out algorithmically—presumably by the cerebellum. At that stage, awareness is no longer necessary, and it becomes safe to allow one’s conscious mind to wander and to contemplate other matters—although, from time to time one may need to check that the algorithm has not been sidetracked in some (perhaps subtle) way.

Penrose, supra note 30, at 413.

210. Id. at 414-16.

211. See generally Hofstadter, Metamagical Themas, supra note 44, at 547-603 (discussing the “natural selection” rationale for justifying analogies); Douglas R. Hofstadter, MIT Artificial Intelligence Lab. Memorandum No. 755, The Copycat Project: An Experiment in Nondeterminism and Creative Analogies (1984) (design for systems capable of doing analogies with the insight and short sight of humans). But see Penrose, supra note 30, at 414 (“I do not see how natural selection, in itself, can evolve algorithms which could have the kind of conscious judgements of the validity of other algorithms that we seem to have.”) (emphasis in original).
V. Conclusion

Gödel's proof reveals that the law cannot be a determinate formal system. Like mathematics, the law is either incomplete and must look outside itself for guidance, or it is inconsistent and contradicts itself. As a mechanical system, the law is and must be indeterminate.

For reasons that have not been made entirely clear, mainstream legal thought has expressed a definite aversion to indeterminacy. We suspect the true reason rests in anxiety over the unknown—the human tendency to fear that which is not understood. In the legal community, this fear has translated itself into a general presumption that anything less than formalism must be nihilistic.

Gödel's Theorem, however, is not a tool of nihilism. It does not disparage the usefulness of algorithms and formal systems; it merely demonstrates their limitations. Following Gödel's discovery in 1931, mathematical studies did not cease; rather, they proceeded at a higher level than ever before. Recognition of the limits of formalism allowed mathematicians (and today's computer scientists) to "see" things never before envisioned. Scholars in these hard sciences consider Gödel's result "an occasion, not for dejection, but for a renewed appreciation of the powers of creative reason."213

Gödel's result, of course, constrains what might be accomplished with the legal system. First, incompleteness and inconsistency reduce the law's reliability and predictability. Because human judgment and intuition are not fungible commodities, achieving certainty or equality of result is impossible. Second, Gödel's Theorem discloses that America's ideal of the "rule of law" can never be fully achieved. Like it or not, the application of legal principles depends on human intuition. Consequently, the American government, as administered through its legal system, is—and can only be—one of people as well as laws. Finally, Gödel's Theorem dilutes any strict vision of representative government. Though elected representatives pass laws, the judiciary applies them. Judicial insight necessarily affects the law's impact on the population, and judicial legislation must be recognized as an inevitable component of government.

As in mathematics, Gödel's result should be welcomed for these revelations. Although an absence of formalism means that law cannot be perfectly reliable, it is the recognition of uncertainty that is the key to an ordered society. Gödel's Theorem allows society to anticipate risk and

212. On the "seeing" process, see Roger Penrose's remarks quoted supra at note 165.
213. NAGEL & NEWMAN, supra note 130, at 102; see id. at 101-02.
plan accordingly. In addition, the deconstruction of legal formalism ought to bring greater individual accountability to government. No longer should officials be heard to hide behind "the law," for the simple reason that the law does not independently exist. Finally, because the facade of pristine republicanism$^{214}$ is removed by Gödel’s result, the judiciary is exposed as a true body politic. "Decisions" and "interpretations" must rely on human choices and hunches—subjective determinations that cannot be quantified.$^{215}$

$^{214}$ The "separation of powers" theory, of course, teaches that the legislature "makes" law, and the judiciary only applies it. See William D. Popkin, Judicial Use of Presidential Legislative History: A Critique, 66 IND. L.J. 699, 699 (1991) (The "broad 'formalist' argument" is "that the 'law' consists of the statute's text as voted on by the legislature and signed by the President.").

$^{215}$ For this reason, formalist suggestions (such as that by Justice Scalia) that a statute's meaning can be gleaned from its text should be taken with a large grain of salt. Justice Scalia has explained his position as follows:

The meaning of terms on the statute books ought to be determined, not on the basis of which meaning can be shown to have been understood by a larger handful of the Members of Congress; but rather on the basis of which meaning is (1) most in accord with context and ordinary usage, and thus most likely to have been understood by the whole Congress which voted on the words of the statute . . . , and (2) most compatible with the surrounding body of law into which the provision must be integrated—a compatibility which, by a benign fiction, we assume Congress always has in mind. I would not permit any of the historical and legislative material discussed by the Court, or all of it combined, to lead me to a result different from the one that these factors suggest.


Scalia's argument ignores the implications of Gödel's Theorem. Because the law is incomplete or inconsistent, no interpreter can be certain in an objective manner that he or she has correctly deciphered the meaning and application of a given proposition. For this reason, excluding any particular source of meaning is at best an arbitrary exercise, and at worst reflects an uncumbered arrogance. Compare Professor D'Amato's view:

The opinions of Justice Scalia urging courts not to use legislative history is in my view a welcome development from the perspective of reducing the legal fees that otherwise would be run up by lawyers on both sides poring over volumes of legislative history. But Justice Scalia has no doctrinal justification for his stance; his preference for "plain meaning" of statutes is no less indeterminate than recourse to legislative history. He has simply traded one indeterminacy for another. The trade is desirable only to the extent that it may reduce transaction costs (legal fees).

D'Amato, Pragmatic Indeterminacy, supra note 9, at 175 n.88.